２年５･６･７組　数学　統一テスト再　～三角関数～　Ｈ２０．１１．１９　　　(　　)組(　　)番　名前(　　　　　　　　　　)

１．$0 \leqq \theta < 2\pi$ のとき，次の方程式または不等式を解け。

(1) $2\cos\theta = -\sqrt{3}$

(2) $\tan\theta = -\frac{1}{\sqrt{3}}$

(3) $2\sin^2\theta - \cos\theta = 2$

(4) $\sin 2\theta = -\cos\theta$

(5) $\sin\left(\theta + \frac{4}{3}\pi\right) = \frac{1}{2}$

(6) $\sin\theta + \sqrt{3}\cos\theta = -\sqrt{3}$

(7) $\cos\theta > -\frac{1}{2}$

(8) $\sin\theta + 2\cos^2\theta < 1$

(9) $\cos 2\theta - \sin\theta \geqq 0$

(10) $\sqrt{3}\sin\theta + \cos\theta \leqq 1$

２．次のような扇形の弧の長さlと面積Sを求めよ。　半径24，中心角 $\frac{7}{24}\pi$

３．次の２直線のなす鋭角を求めよ。（ただし弧度法を用いて表すこと）

$x + y = 0$，$y = (\sqrt{3} - 2)x$

4．αは鈍角，βは鋭角とする。次の値を求めよ。

$\sin\alpha=\dfrac{2}{3}$, $\cos\beta=\dfrac{1}{5}$のとき，$\sin(\alpha+\beta)$, $\cos(\alpha+\beta)$

5．次の関数の最大値･最小値とそのときのθの値を求めよ。

$y=2\sin\left(\theta-\dfrac{\pi}{6}\right)+1$ $(0\leqq\theta\leqq\pi)$

6．$0\leqq\theta<2\pi$のとき，関数$y=-\cos2\theta-2\cos\theta+3$の最大値･最小値とそのときの$\theta$の値を求めよ。

2年5・6・7組　数学　統一テスト再　～三角関数～　H２０．１１．１９　　1 各6点×10　2～6 各8点×5　　（　）組（　）番　名前（　こたえ　）

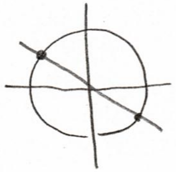
1．$0 \leqq \theta < 2\pi$ のとき，次の方程式または不等式を解け。

(1) $2\cos\theta = -\sqrt{3}$

$\cos\theta = -\frac{\sqrt{3}}{2}$

$\therefore\ \theta = \frac{5}{6}\pi,\ \frac{7}{6}\pi$

(2) $\tan\theta = -\frac{1}{\sqrt{3}}$



$\theta = \frac{5}{6}\pi,\ \frac{11}{6}\pi$

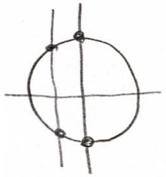
(3) $2\sin^2\theta - \cos\theta = 2$

$2(1-\cos^2\theta) - \cos\theta - 2 = 0$

$\therefore\ 2\cos^2\theta + \cos\theta = 0$

$\cos\theta(2\cos\theta + 1) = 0$

$\therefore\ \cos\theta = 0,\ -\frac{1}{2}$



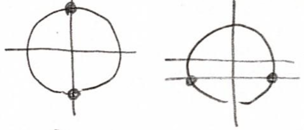
$\theta = \frac{\pi}{2},\ \frac{2}{3}\pi,\ \frac{4}{3}\pi,\ \frac{3}{2}\pi$

(4) $\sin 2\theta = -\cos\theta$

$2\sin\theta\cos\theta = -\cos\theta$

$\cos\theta(2\sin\theta + 1) = 0$

$\therefore\ \cos\theta = 0,\ \sin\theta = -\frac{1}{2}$

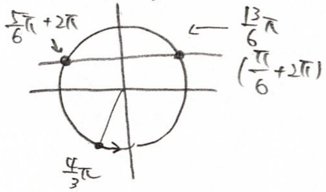


$\therefore\ \theta = \frac{\pi}{2},\ \frac{3}{2}\pi,\ \frac{7}{6}\pi,\ \frac{11}{6}\pi$

(5) $\sin\left(\theta + \frac{4}{3}\pi\right) = \frac{1}{2}$

$0 \leqq \theta < 2\pi$ より

$\frac{4}{3}\pi \leqq \theta + \frac{4}{3}\pi < 2\pi + \frac{4}{3}\pi$

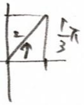


$\therefore\ \theta + \frac{4}{3}\pi = \frac{13}{6}\pi,\ \frac{17}{6}\pi$

$\therefore\ \theta = \frac{5}{6}\pi,\ \frac{3}{2}\pi$

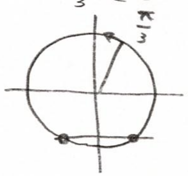
(6) $\sin\theta + \sqrt{3}\cos\theta = -\sqrt{3}$

$2\sin\left(\theta + \frac{\pi}{3}\right) = -\sqrt{3}$



$\sin\left(\theta + \frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$

$\frac{\pi}{3} \leqq \theta + \frac{\pi}{3} < 2\pi + \frac{\pi}{3}$



$\therefore\ \theta + \frac{\pi}{3} = \frac{4}{3}\pi,\ \frac{5}{3}\pi$

$\therefore\ \theta = \pi,\ \frac{4}{3}\pi$

(7) $\cos\theta > -\frac{1}{2}$

$\therefore$

$0 \leqq \theta < \frac{2}{3}\pi$

$\frac{4}{3}\pi < \theta < 2\pi$

(8) $\sin\theta + 2\cos^2\theta < 1$

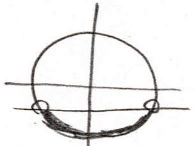
$\sin\theta + 2(1 - \sin^2\theta) < 1$

$\therefore$

$2\sin^2\theta - \sin\theta - 1 > 0$

$(2\sin\theta + 1)(\sin\theta - 1) > 0$

$\therefore\ \sin\theta < -\frac{1}{2},\ \sin\theta > 1$



$\therefore$

$\frac{7}{6}\pi < \theta < \frac{11}{6}\pi$

(9) $\cos 2\theta - \sin\theta \geqq 0$

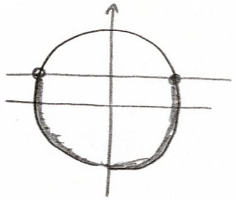
$1 - 2\sin^2\theta - \sin\theta \geqq 0$

$2\sin^2\theta + \sin\theta - 1 \leqq 0$

$(2\sin\theta - 1)(\sin\theta + 1) \leqq 0$

$\therefore$

$-1 \leqq \sin\theta \leqq \frac{1}{2}$



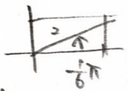
$\therefore$

$0 \leqq \theta \leqq \frac{1}{6}\pi$

$\frac{5}{6}\pi \leqq \theta < 2\pi$

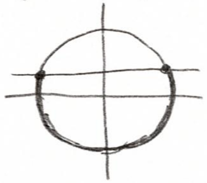
(10) $\sqrt{3}\sin\theta + \cos\theta \leqq 1$

$2\sin\left(\theta + \frac{1}{6}\pi\right) \leqq 1$



$\sin\left(\theta + \frac{1}{6}\pi\right) \leqq \frac{1}{2}$

$\frac{1}{6}\pi \leqq \theta + \frac{1}{6}\pi < 2\pi + \frac{1}{6}\pi$



$\therefore$

$\theta + \frac{\pi}{6} = \frac{\pi}{6},$

$\frac{5}{6}\pi \leqq \theta + \frac{\pi}{6} < 2\pi + \frac{1}{6}\pi$

$\therefore$

$\theta = 0,$

$\frac{2}{3}\pi \leqq \theta < 2\pi$

2．次のような扇形の弧の長さlと面積Sを求めよ。　半径24，中心角 $\frac{7}{24}\pi$

$l = r\theta = 24 \cdot \frac{7}{24}\pi = 7\pi$

$S = \frac{1}{2}rl = \frac{1}{2} \cdot 24 \cdot 7\pi = 12 \cdot 7\pi$

$= 84\pi$

3．次の２直線のなす鋭角を求めよ。（ただし弧度法を用いて表すこと）

$x + y = 0$，$y = (\sqrt{3} - 2)x$

$y = -x$, $y = (\sqrt{3} - 2)x$ と x軸の正の方向となす角度をそれぞれ α, β とすると，求める角度 θ は

$\tan\theta = |\tan(\alpha - \beta)|$

$= \left|\frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}\right|$

が成り立つ。ここで，$\tan\alpha = -1$，$\tan\beta = \sqrt{3} - 2$ より

$\tan\theta = \left|\frac{-1 - (\sqrt{3} - 2)}{1 + (-1)(\sqrt{3} - 2)}\right|$

$= \left|\frac{-1 - \sqrt{3} + 2}{1 - \sqrt{3} + 2}\right|$

$= \left|\frac{1 - \sqrt{3}}{3 - \sqrt{3}}\right|$

$= \left|\frac{-(\sqrt{3} - 1)}{\sqrt{3}(\sqrt{3} - 1)}\right|$

$= \left|-\frac{1}{\sqrt{3}}\right|$

$= \frac{1}{\sqrt{3}}$

$\therefore$

$\theta = \frac{1}{6}\pi$

4．αは鈍角，βは鋭角とする。次の値を求めよ。

$\sin\alpha = \frac{2}{3}$, $\cos\beta = \frac{1}{5}$ のとき，$\sin(\alpha+\beta)$, $\cos(\alpha+\beta)$

$\sin^2\alpha + \cos^2\alpha = 1$ より

$\cos^2\alpha = 1 - \sin^2\alpha = 1 - \left(\frac{2}{3}\right)^2 = \frac{5}{9}$

αは鈍角より $\cos\alpha < 0$ より $\cos\alpha = -\frac{\sqrt{5}}{3}$

$\sin^2\beta + \cos^2\beta = 1$ より

$\sin^2\beta = 1 - \cos^2\beta = 1 - \left(\frac{1}{5}\right)^2 = \frac{24}{25}$

βは鋭角より $\sin\beta > 0$ から $\sin\beta = \frac{2\sqrt{6}}{5}$

∴

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$= \frac{2}{3}\cdot\frac{1}{5} + \left(-\frac{\sqrt{5}}{3}\right)\cdot\frac{2\sqrt{6}}{5}$

$= \frac{2 - 2\sqrt{30}}{15}$ //

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

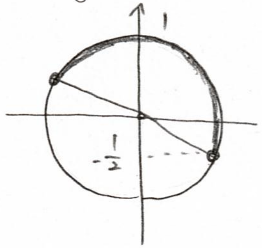
$= \left(-\frac{\sqrt{5}}{3}\right)\cdot\frac{1}{5} - \frac{2}{3}\cdot\frac{2\sqrt{6}}{5}$

$= -\frac{\sqrt{5} + 4\sqrt{6}}{15}$ //

5．次の関数の最大値・最小値とそのときのθの値を求めよ。

$y = 2\sin\left(\theta - \frac{\pi}{6}\right) + 1$ $(0 \leqq \theta \leqq \pi)$

$-\frac{\pi}{6} \leqq \theta - \frac{\pi}{6} \leqq \frac{5}{6}\pi$



∴ $-\frac{1}{2} \leqq \sin\left(\theta - \frac{\pi}{6}\right) \leqq 1$

$-1 \leqq 2\sin\left(\theta - \frac{\pi}{6}\right) \leqq 2$

∴ $0 \leqq 2\sin\left(\theta - \frac{\pi}{6}\right) + 1 \leqq 3$

よって 最大値3，最小値0

最大は $\theta - \frac{\pi}{6} = \frac{1}{2}\pi$ ∴ $\theta = \frac{2}{3}\pi$

最小は $\theta - \frac{\pi}{6} = -\frac{1}{6}\pi$ ∴ $\theta = 0$

以上より

最大値 3 $\left(\theta = \frac{2}{3}\pi\right)$

最小値 0 $(\theta = 0)$ //

6．$0 \leqq \theta < 2\pi$のとき，関数$y = -\cos 2\theta - 2\cos\theta + 3$の最大値・最小値とそのときの$\theta$の値を求めよ。

$y = -(2\cos^2\theta - 1) - 2\cos\theta + 3$

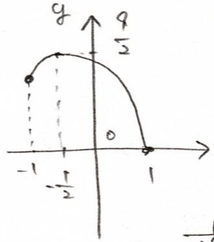
$= -2\cos^2\theta - 2\cos\theta + 4$

$= -2(\cos^2\theta + \cos\theta) + 4$

$= -2\left\{\left(\cos\theta + \frac{1}{2}\right)^2 - \frac{1}{4}\right\} + 4$

$= -2\left(\cos\theta + \frac{1}{2}\right)^2 + \frac{9}{2}$

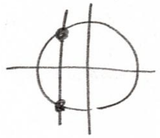
$0 \leqq \theta < 2\pi$ より $-1 \leqq \cos\theta \leqq 1$



最大値 $\frac{9}{2}$

最小値 0

最大は $\cos\theta = -\frac{1}{2}$ より



$\theta = \frac{2}{3}\pi$, $\frac{4}{3}\pi$

最小は $\cos\theta = 1$ より

$\theta = 0$

∴

最大値 $\frac{9}{2}$ $\left(\theta = \frac{2}{3}\pi, \frac{4}{3}\pi\right)$

最大値 0 $(\theta = 0)$ //