

1. 次の式を因数分解せよ。

(1) $9x^2 + 12x + 4$

(2) $a^2 - 6a + 5$

(3) $x^2 - 4xy - 12y^2$

(4) $3a^2 + 10a + 3$

(5) $6p^2 - 5p + 1$

(6) $x^3 - 27y^3$

(7) $x^3y^3 - z^3$

(8) $x^2 - (a+1)x + a$

(9) $2x^2 + (3a-10)x - 15a$

(10) $a^2 + 5ab + 6b^2 + a + b - 2$

(11) $x^3 + y^3 - 3xy + 1$

(12) $x^4 - y^4$

(13) $a^4 - 8a^2 - 9$

(14) $-x^6 + y^6$

(15) $x^3 + ax + a + 1$

(16) $ab(a-b) + bc(b-c) + ca(c-a)$

(17) $(x^2 + 4x)^2 - 9(x^2 + 4x) - 36$

(18) $(x-3)(x-1)(x+3)(x+5) + 35$

(19) $a^2b + ab^2 + b^2c + bc^2 + c^2a + ca^2 + 2abc$

(20) $a^2(b-c) + b^2(c-a) + c^2(a-b)$

1. 次の式を因数分解せよ。

(1) $9x^2 + 12x + 4$

$$= (3x+2)^2$$

(3) $x^2 - 4xy - 12y^2$

$$= (x+2y)(x-6y)$$

(5) $6p^2 - 5p + 1$

$$= (3p-1)(2p-1)$$

(7) $x^3y^3 - z^3$

$$= (xy-z)(x^2y^2 + xyz + z^2)$$

(9) $2x^2 + (3a-10)x - 15a$

$$= (3x-15)a + (2x^2-10x) \\ = 3(x-5)a + 2x(x-5) \\ = (x-5)(3a+2x) \\ = (x-5)(2x+3a)$$

(2) $a^2 - 6a + 5$

$$= (a-1)(a-5)$$

(4) $3a^2 + 10a + 3$

$$= (3a+1)(a+3)$$

(6) $x^3 - 27y^3$

$$= (x-3y)(x^2+3xy+9y^2)$$

(8) $x^2 - (a+1)x + a$

$$= (x-a)(x-1)$$

(10) $a^2 + 5ab + 6b^2 + a + b - 2$

$$= a^2 + (5b+1)a + (6b^2+b-2) \\ = a^2 + (5b+1)a + (3b+2)(2b-1) \\ = \{a+(3b+2)\} \{a+(2b-1)\} \\ = (a+3b+2)(a+2b-1)$$

(11) $x^3 + y^3 - 3xy + 1$

$$= (x+y)^3 - 3xy(x+y) - 3xy + 1 \\ = (x+y)^2 + 1 - 3xy(x+y+1) \\ = (x+y+1)\{(x+y)^2 - (x+y) + 1\} - 3xy(x+y+1) \\ = (x+y+1)\{x^2+y^2 - (x+y) + 1 - 3xy\} \\ = (x+y+1)(x^2+y^2+1-x-y-xy)$$

(13) $a^4 - 8a^2 - 9$

$$= (a^2-9)(a^2+1) \\ = (a+3)(a-3)(a^2+1)$$

(15) $x^3 + ax + a + 1$

$$= (x+1)a + (x^3+1) \\ = (x+1)a + (x+1)(x^2-x+1) \\ = (x+1)\{a + (x^2-x+1)\} \\ = (x+1)(x^2-x+a+1)$$

(17) $(x^2+4x)^2 - 9(x^2+4x) - 36$

$$= \{(x^2+4x)-12\} \{(x^2+4x)+3\} \\ = (x^2+4x-12)(x^2+4x+3) \\ = (x+6)(x-2)(x+1)(x+3)$$

(19) $a^2b + ab^2 + b^2c + bc^2 + c^2a + ca^2 + 2abc$

$$= (b+c)a^2 + (b^2+c^2+2bc)a + (b^2c+bc^2) \\ = (b+c)a^2 + (b+c)^2a + bc(b+c) \\ = (b+c)\{a^2 + (b+c)a + bc\} \\ = (b+c)(a+b)(a+c) \\ = (a+b)(b+c)(c+a)$$

(12) $x^4 - y^4$

$$= (x^2-y^2)(x^2+y^2) \\ = (x+y)(x-y)(x^2+y^2)$$

(14) $-x^6 + y^6$

$$= -(x^6 - y^6) \\ = -(x^3+y^3)(x^3-y^3) \\ = -(x+y)(x^2-xy+y^2)(x-y)(x^2+xy+y^2)$$

(16) $ab(a-b) + bc(b-c) + ca(c-a)$

$$= a^2b - ab^2 + b^2c - bc^2 + c^2a - ca^2 \\ = (b-c)a^2 - (b^2-c^2)a + (b^2c-bc^2) \\ = (b-c)a^2 - (b-c)(b+c)a + bc(b-c) \\ = (b-c)\{a^2 - (b+c)a + bc\} \\ = (b-c)(a-b)(a-c) = -(a-b)(b-c)(c-a)$$

(18) $(x-3)(x-1)(x+3)(x+5) + 35$

$$= (x^2+2x-15)(x^2+2x-3) + 35 \quad x^2+2x=A \\ = (A-15)(A-3) + 35 \\ = A^2 - 18A + 80 \\ = (A-8)(A-10) \\ = (x^2+2x-8)(x^2+2x-10) \\ = (x+4)(x-2)(x^2+2x-10)$$

(20) $a^2(b-c) + b^2(c-a) + c^2(a-b)$

$$= (b-c)a^2 - (b^2+c^2)a + (b^2c-bc^2) \\ = (b-c)a^2 - (b-c)(b+c)a + bc(b-c) \\ = (b-c)\{a^2 - (b+c)a + bc\} \\ = (b-c)(a-b)(a-c) \\ = -(a-b)(b-c)(c-a)$$