

分数式クイズ

1 次の分数式を約分して簡単にせよ。

(1) $\frac{8a^2x^3y}{40axy^2}$

(2) $\frac{x^2+6x+5}{x^2-2x-3}$

(3) $\frac{x^2-1}{x^3-1}$

(4) $\frac{a^3+3a^2b-4ab^2}{2a^2-4ab+2b^2}$

解答 (1) $\frac{ax^2}{5y}$ (2) $\frac{x+5}{x-3}$ (3) $\frac{x+1}{x^2+x+1}$ (4) $\frac{a(a+4b)}{2(a-b)}$

解説

(1) $\frac{8a^2x^3y}{40axy^2}=\frac{ax^2}{5y}$

(2) $\frac{x^2+6x+5}{x^2-2x-3}=\frac{(x+1)(x+5)}{(x+1)(x-3)}=\frac{x+5}{x-3}$

(3) $\frac{x^2-1}{x^3-1}=\frac{(x+1)(x-1)}{(x-1)(x^2+x+1)}=\frac{x+1}{x^2+x+1}$

(4) $\frac{a^3+3a^2b-4ab^2}{2a^2-4ab+2b^2}=\frac{a(a^2+3ab-4b^2)}{2(a^2-2ab+b^2)}=\frac{a(a-b)(a+4b)}{2(a-b)^2}=\frac{a(a+4b)}{2(a-b)}$

2 次の式を計算せよ。

(1) $\frac{x^2-xy-6y^2}{x^2+xy}\times\frac{x^2-y^2}{x^2-5xy+6y^2}$

(2) $\frac{x^2-x}{x^2-7x+12}\div\frac{x^2+5x}{x^2+2x-15}$

解答 (1) $\frac{(x+2y)(x-y)}{x(x-2y)}$ (2) $\frac{x-1}{x-4}$

解説

(1) 与式 $=\frac{(x+2y)(x-3y)}{x(x+y)}\times\frac{(x+y)(x-y)}{(x-2y)(x-3y)}=\frac{(x+2y)(x-y)}{x(x-2y)}$

(2) 与式 $=\frac{x^2-x}{x^2-7x+12}\times\frac{x^2+2x-15}{x^2+5x}=\frac{x(x-1)}{(x-3)(x-4)}\times\frac{(x-3)(x+5)}{x(x+5)}=\frac{x-1}{x-4}$

3 次の式を計算せよ。

(1) $\frac{x^2-2}{x+2}-\frac{2}{x+2}$

(2) $\frac{2x}{x^2-a^2}+\frac{2a}{a^2-x^2}$

解答 (1) $x-2$ (2) $\frac{2}{x+a}$

解説

(1) 与式 $=\frac{(x^2-2)-2}{x+2}=\frac{x^2-4}{x+2}=\frac{(x+2)(x-2)}{x+2}=x-2$

(2) 与式 $=\frac{2x}{x^2-a^2}-\frac{2a}{x^2-a^2}=\frac{2x-2a}{x^2-a^2}=\frac{2(x-a)}{(x+a)(x-a)}=\frac{2}{x+a}$

4 次の式を計算せよ。

(1) $\frac{1}{x^2+x}+\frac{1}{x^2+3x+2}$

(2) $\frac{2x-3}{x^2-3x+2}-\frac{3x-2}{x^2-4}$

解答 (1) $\frac{2}{x(x+2)}$ (2) $-\frac{x-4}{(x-1)(x+2)}$

解説

(1) 与式 $=\frac{1}{x(x+1)}+\frac{1}{(x+1)(x+2)}=\frac{x+2}{x(x+1)(x+2)}+\frac{x}{x(x+1)(x+2)}$
 $=\frac{(x+2)+x}{x(x+1)(x+2)}=\frac{2x+2}{x(x+1)(x+2)}=\frac{2(x+1)}{x(x+1)(x+2)}=\frac{2}{x(x+2)}$

(2) 与式 $=\frac{2x-3}{(x-1)(x-2)}-\frac{3x-2}{(x+2)(x-2)}=\frac{(2x-3)(x+2)-(3x-2)(x-1)}{(x-1)(x-2)(x+2)}$
 $=\frac{(2x^2+x-6)-(3x^2-5x+2)}{(x-1)(x-2)(x+2)}=\frac{-(x^2-6x+8)}{(x-1)(x-2)(x+2)}$
 $=-\frac{(x-2)(x-4)}{(x-1)(x-2)(x+2)}=-\frac{x-4}{(x-1)(x+2)}$

5 次の式を計算せよ。

(1) $\frac{x^2+2x-3}{x^2-x}\times\frac{x^2-2x}{x^2+x-6}$

(2) $\frac{9x^2-4y^2}{6x^2+xy-2y^2}\div\frac{6x^2-xy-2y^2}{4x^2-4xy+y^2}$

(3) $\frac{x-3}{x^2+3x+2}-\frac{x-4}{x^2+2x+1}$

(4) $\frac{2}{4x^2-1}+\frac{3x}{2x^2-x-1}$

解答 (1) 1 (2) $\frac{2x-y}{2x+y}$ (3) $\frac{5}{(x+1)^2(x+2)}$ (4) $\frac{3x-2}{(2x-1)(x-1)}$

解説

(1) 与式 $=\frac{(x-1)(x+3)}{x(x-1)}\times\frac{x(x-2)}{(x-2)(x+3)}=1$

(2) 与式 $=\frac{9x^2-4y^2}{6x^2+xy-2y^2}\times\frac{4x^2-4xy+y^2}{6x^2-xy-2y^2}$
 $=\frac{(3x+2y)(3x-2y)}{(3x+2y)(2x-y)}\times\frac{(2x-y)^2}{(3x-2y)(2x+y)}=\frac{2x-y}{2x+y}$

(3) 与式 $=\frac{x-3}{(x+1)(x+2)}-\frac{x-4}{(x+1)^2}=\frac{(x-3)(x+1)-(x-4)(x+2)}{(x+1)^2(x+2)}$
 $=\frac{(x^2-2x-3)-(x^2-2x-8)}{(x+1)^2(x+2)}=\frac{5}{(x+1)^2(x+2)}$

(4) 与式 $=\frac{2}{(2x+1)(2x-1)}+\frac{3x}{(x-1)(2x+1)}=\frac{2(x-1)+3x(2x-1)}{(2x+1)(2x-1)(x-1)}$
 $=\frac{6x^2-x-2}{(2x+1)(2x-1)(x-1)}=\frac{(2x+1)(3x-2)}{(2x+1)(2x-1)(x-1)}=\frac{3x-2}{(2x-1)(x-1)}$

6 $\frac{x^2-6x+9}{x^2-10x+21}+\frac{x+1}{x-7}$ を簡単にせよ。[10点]

解答 与式 $=\frac{(x-3)^2}{(x-3)(x-7)}+\frac{x+1}{x-7}=\frac{x-3}{x-7}+\frac{x+1}{x-7}=\frac{2x-2}{x-7}$

解説

与式 $=\frac{(x-3)^2}{(x-3)(x-7)}+\frac{x+1}{x-7}=\frac{x-3}{x-7}+\frac{x+1}{x-7}=\frac{2x-2}{x-7}$

7 (1) 次の分数式を約分して、既約分数式にせよ。

(ア) $\frac{9ax^2y}{18a^3xy^2}$

(イ) $\frac{x^2-4x+3}{2x^2-2x-12}$

(2) 次の計算をせよ。

(ア) $\frac{2xy}{3ab}\times\frac{9a^2b^3}{8x^3y}$

(イ) $\frac{x^2+2x}{x^2+4x+3}\times\frac{x+3}{x^2+x-2}\div\frac{x+1}{x-1}$

解答 (1) (ア) $\frac{x}{2a^2y}$ (イ) $\frac{x-1}{2(x+2)}$ (2) (ア) $\frac{3ab^2}{4x^2}$ (イ) $\frac{x}{(x+1)^2}$

解説

(1) (ア) $\frac{9ax^2y}{18a^3xy^2}=\frac{x}{2a^2y}$

(イ) $\frac{x^2-4x+3}{2x^2-2x-12}=\frac{(x-1)(x-3)}{2(x+2)(x-3)}=\frac{x-1}{2(x+2)}$

(2) (ア) $\frac{2xy}{3ab}\times\frac{9a^2b^3}{8x^3y}=\frac{2xy\times9a^2b^3}{3ab\times8x^3y}=\frac{3ab^2}{4x^2}$

(イ) $\frac{x^2+2x}{x^2+4x+3}\times\frac{x+3}{x^2+x-2}\div\frac{x+1}{x-1}=\frac{x(x+2)}{(x+1)(x+3)}\times\frac{x+3}{(x-1)(x+2)}\times\frac{x-1}{x+1}$
 $=\frac{x}{(x+1)^2}$

8 (1) 次の分数式を約分して、既約分数式にせよ。

(ア) $\frac{4a^2bc^3}{12ab^3c}$

(イ) $\frac{a^4+a^3-2a^2}{a^2-4}$

(ウ) $\frac{x^4-y^4}{(x-y)(x^3+y^3)}$

(2) 次の計算をせよ。

(ア) $\frac{8x^3z}{9bc^3}\times\frac{27abc}{4xyz^2}$

(イ) $\frac{a+b}{a^2+b^2}\times\frac{a^3+ab^2}{a^2-b^2}$

(ウ) $\frac{x^2+5x+4}{x^2+2x}\div\frac{x+4}{x}\times\frac{1}{x+1}$

解答 (1) (ア) $\frac{ac^2}{3b^2}$ (イ) $\frac{a^2(a-1)}{a-2}$ (ウ) $\frac{x^2+y^2}{x^2-xy+y^2}$

(2) (ア) $\frac{6ax^2}{c^2yz}$ (イ) $\frac{a}{a-b}$ (ウ) $\frac{1}{x+2}$

解説

(1) (ア) $\frac{4a^2bc^3}{12ab^3c}=\frac{4abc\cdot ac^2}{4abc\cdot 3b^2}=\frac{ac^2}{3b^2}$

(イ) $\frac{a^4+a^3-2a^2}{a^2-4}=\frac{a^2(a^2+a-2)}{(a+2)(a-2)}=\frac{a^2(a+2)(a-1)}{(a+2)(a-2)}$
 $=\frac{a^2(a-1)}{a-2}$

(ウ) $\frac{x^4-y^4}{(x-y)(x^3+y^3)}=\frac{(x^2-y^2)(x^2+y^2)}{(x-y)(x+y)(x^2-xy+y^2)}$
 $=\frac{(x-y)(x+y)(x^2+y^2)}{(x-y)(x+y)(x^2-xy+y^2)}$
 $=\frac{x^2+y^2}{x^2-xy+y^2}$

(2) (ア) (与式) $=\frac{2^3\cdot 3^3}{2^2\cdot 3^2}\times a\times\frac{b}{b}\times\frac{c}{c^3}\times\frac{x^3}{x}\times\frac{1}{y}\times\frac{z}{z^2}=\frac{6ax^2}{c^2yz}$

(イ) (与式) $=\frac{a+b}{a^2+b^2}\times\frac{a(a^2+b^2)}{(a+b)(a-b)}=\frac{a}{a-b}$

(ウ) (与式) $=\frac{(x+1)(x+4)}{x(x+2)}\times\frac{x}{x+4}\times\frac{1}{x+1}=\frac{1}{x+2}$

9 次の計算をせよ。

(1) $\frac{x+1}{x^2+2x-3}-\frac{x}{x^2-9}$

(2) $\frac{4}{x^2+4}-\frac{1}{x-2}+\frac{1}{x+2}$

解答 (1) $-\frac{1}{(x-1)(x-3)}$ (2) $-\frac{32}{x^4-16}$

解説

(1) $\frac{x+1}{x^2+2x-3}-\frac{x}{x^2-9}=\frac{x+1}{(x-1)(x+3)}-\frac{x}{(x+3)(x-3)}$
 $=\frac{(x+1)(x-3)}{(x-1)(x+3)(x-3)}-\frac{x(x-1)}{(x-1)(x+3)(x-3)}$
 $=\frac{(x+1)(x-3)-x(x-1)}{(x-1)(x+3)(x-3)}=\frac{-(x+3)}{(x-1)(x+3)(x-3)}$
 $=-\frac{1}{(x-1)(x-3)}$

$$\begin{aligned}
(2) \quad & \frac{4}{x^2+4} - \frac{1}{x-2} + \frac{1}{x+2} = \frac{4}{x^2+4} - \left(\frac{1}{x-2} - \frac{1}{x+2} \right) \\
&= \frac{4}{x^2+4} - \frac{(x+2)-(x-2)}{(x-2)(x+2)} \\
&= \frac{4}{x^2+4} - \frac{4}{x^2-4} \\
&= \frac{4\{x^2-4-(x^2+4)\}}{(x^2+4)(x^2-4)} = \frac{4 \cdot (-8)}{(x^2)^2-4^2} \\
&= -\frac{32}{x^4-16}
\end{aligned}$$

10 次の計算をせよ。

$$(1) \quad \frac{2x+7}{x^2+6x+8} - \frac{x-4}{x^2-4} \qquad (2) \quad \frac{a+b}{a-b} + \frac{a-b}{a+b} - \frac{2(a^2-b^2)}{a^2+b^2}$$

解答 (1) $\frac{x+1}{(x-2)(x+4)}$ (2) $\frac{8a^2b^2}{a^4-b^4}$

解説

$$\begin{aligned}
(1) \quad & (\text{与式}) = \frac{2x+7}{(x+2)(x+4)} - \frac{x-4}{(x+2)(x-2)} \\
&= \frac{(2x+7)(x-2)}{(x+2)(x-2)(x+4)} - \frac{(x-4)(x+4)}{(x+2)(x-2)(x+4)} \\
&= \frac{(2x+7)(x-2)-(x-4)(x+4)}{(x+2)(x-2)(x+4)} \\
&= \frac{x^2+3x+2}{(x+2)(x-2)(x+4)} = \frac{(x+1)(x+2)}{(x+2)(x-2)(x+4)} \\
&= \frac{x+1}{(x-2)(x+4)} \\
(2) \quad & (\text{与式}) = \frac{(a+b)^2+(a-b)^2}{(a-b)(a+b)} - \frac{2(a^2-b^2)}{a^2+b^2} \\
&= \frac{2(a^2+b^2)}{a^2-b^2} - \frac{2(a^2-b^2)}{a^2+b^2} \\
&= \frac{2\{(a^2+b^2)^2-(a^2-b^2)^2\}}{(a^2-b^2)(a^2+b^2)} = \frac{8a^2b^2}{a^4-b^4}
\end{aligned}$$

11 次の計算をせよ。

$$(1) \quad \frac{1}{b-a} \left(\frac{1}{x+a} - \frac{1}{x+b} \right) \\
(2) \quad \frac{1}{(x+1)(x+3)} + \frac{1}{(x+3)(x+5)} + \frac{1}{(x+5)(x+7)}$$

解答 (1) $\frac{1}{(x+a)(x+b)}$ (2) $\frac{3}{(x+1)(x+7)}$

解説

$$\begin{aligned}
(1) \quad & \frac{1}{b-a} \left(\frac{1}{x+a} - \frac{1}{x+b} \right) = \frac{1}{b-a} \cdot \frac{(x+b)-(x+a)}{(x+a)(x+b)} \\
&= \frac{1}{b-a} \cdot \frac{b-a}{(x+a)(x+b)} \\
&= \frac{1}{(x+a)(x+b)} \\
(2) \quad & \frac{1}{(x+1)(x+3)} + \frac{1}{(x+3)(x+5)} + \frac{1}{(x+5)(x+7)} \\
&= \frac{1}{2} \left(\frac{1}{x+1} - \frac{1}{x+3} \right) + \frac{1}{2} \left(\frac{1}{x+3} - \frac{1}{x+5} \right) + \frac{1}{2} \left(\frac{1}{x+5} - \frac{1}{x+7} \right) \\
&= \frac{1}{2} \left(\frac{1}{x+1} - \frac{1}{x+7} \right) = \frac{1}{2} \cdot \frac{x+7-(x+1)}{(x+1)(x+7)} = \frac{3}{(x+1)(x+7)}
\end{aligned}$$

12 次の計算をせよ。

$$(1) \quad \frac{1}{(x-1)x} + \frac{1}{x(x+1)} + \frac{1}{(x+1)(x+2)} \qquad (2) \quad \frac{2}{(n-2)n} + \frac{2}{n(n+2)} + \frac{2}{(n+2)(n+4)}$$

解答 (1) $\frac{3}{(x-1)(x+2)}$ (2) $\frac{6}{(n-2)(n+4)}$

解説

$$\begin{aligned}
(1) \quad & (\text{与式}) = \left(\frac{1}{x-1} - \frac{1}{x} \right) + \left(\frac{1}{x} - \frac{1}{x+1} \right) + \left(\frac{1}{x+1} - \frac{1}{x+2} \right) \\
&= \frac{1}{x-1} - \frac{1}{x+2} = \frac{3}{(x-1)(x+2)} \\
(2) \quad & (\text{与式}) = \left(\frac{1}{n-2} - \frac{1}{n} \right) + \left(\frac{1}{n} - \frac{1}{n+2} \right) + \left(\frac{1}{n+2} - \frac{1}{n+4} \right) \\
&= \frac{1}{n-2} - \frac{1}{n+4} = \frac{6}{(n-2)(n+4)}
\end{aligned}$$

13 次の計算をせよ。

$$(1) \quad \frac{x^2+4x+5}{x+3} - \frac{x^2+5x+6}{x+4} \qquad (2) \quad \frac{x+4}{x+2} - \frac{x+5}{x+1} - \frac{x-5}{x-1} + \frac{x-4}{x-2}$$

解答 (1) $\frac{2}{(x+3)(x+4)}$ (2) $-\frac{24}{(x+2)(x-2)(x+1)(x-1)}$

解説

$$\begin{aligned}
(1) \quad & \frac{x^2+4x+5}{x+3} - \frac{x^2+5x+6}{x+4} = \frac{(x+3)(x+1)+2}{x+3} - \frac{(x+4)(x+1)+2}{x+4} \\
&= \left(x+1 + \frac{2}{x+3} \right) - \left(x+1 + \frac{2}{x+4} \right) \\
&= \frac{2}{x+3} - \frac{2}{x+4} = \frac{2\{(x+4)-(x+3)\}}{(x+3)(x+4)} \\
&= \frac{2}{(x+3)(x+4)} \\
(2) \quad & \frac{x+4}{x+2} - \frac{x+5}{x+1} - \frac{x-5}{x-1} + \frac{x-4}{x-2} \\
&= \left(1 + \frac{2}{x+2} \right) - \left(1 + \frac{4}{x+1} \right) - \left(1 - \frac{4}{x-1} \right) + \left(1 - \frac{2}{x-2} \right) \\
&= \frac{2}{x+2} - \frac{4}{x+1} + \frac{4}{x-1} - \frac{2}{x-2} = 2 \left(\frac{1}{x+2} - \frac{1}{x-2} \right) - 4 \left(\frac{1}{x+1} - \frac{1}{x-1} \right) \\
&= \frac{2\{(x-2)-(x+2)\}}{(x+2)(x-2)} - \frac{4\{(x-1)-(x+1)\}}{(x+1)(x-1)} = \frac{-8}{(x+2)(x-2)} + \frac{8}{(x+1)(x-1)} \\
&= \frac{8\{-(x+1)(x-1)+(x+2)(x-2)\}}{(x+2)(x-2)(x+1)(x-1)} = -\frac{24}{(x+2)(x-2)(x+1)(x-1)}
\end{aligned}$$

14 次の計算をせよ。

$$(1) \quad \frac{x^2+2x+3}{x} - \frac{x^2+3x+5}{x+1} \qquad (2) \quad \frac{x+1}{x+2} - \frac{x+2}{x+3} - \frac{x+3}{x+4} + \frac{x+4}{x+5}$$

解答 (1) $\frac{3}{x(x+1)}$ (2) $-\frac{2(2x+7)}{(x+2)(x+3)(x+4)(x+5)}$

解説

$$\begin{aligned}
(1) \quad & (\text{与式}) = \frac{x(x+2)+3}{x} - \frac{(x+1)(x+2)+3}{x+1} \\
&= \left(x+2 + \frac{3}{x} \right) - \left(x+2 + \frac{3}{x+1} \right) \\
&= \frac{3}{x} - \frac{3}{x+1} = \frac{3\{(x+1)-x\}}{x(x+1)} = \frac{3}{x(x+1)} \\
(2) \quad & (\text{与式}) = \left(1 - \frac{1}{x+2} \right) - \left(1 - \frac{1}{x+3} \right) - \left(1 - \frac{1}{x+4} \right) + \left(1 - \frac{1}{x+5} \right)
\end{aligned}$$

$$\begin{aligned}
&= -\frac{1}{x+2} + \frac{1}{x+3} + \frac{1}{x+4} - \frac{1}{x+5} \\
&= \frac{-(x+3)+(x+2)}{(x+2)(x+3)} + \frac{(x+5)-(x+4)}{(x+4)(x+5)} \\
&= \frac{-1}{(x+2)(x+3)} + \frac{1}{(x+4)(x+5)} \\
&= \frac{-(x+4)(x+5)+(x+2)(x+3)}{(x+2)(x+3)(x+4)(x+5)} \\
&= -\frac{2(2x+7)}{(x+2)(x+3)(x+4)(x+5)}
\end{aligned}$$

15 次の式を簡単にせよ。

$$(1) \quad \frac{x - \frac{1}{x}}{\frac{2}{x+1} - \frac{1}{x}} \qquad (2) \quad \frac{1}{1 - \frac{1}{1 - \frac{2}{2+a}}}$$

解答 (1) $(x+1)^2$ (2) $-\frac{a}{2}$

解説

$$\begin{aligned}
(1) \quad & \frac{2}{x+1} - \frac{1}{x} = \frac{2x-(x+1)}{x(x+1)} = \frac{x-1}{x(x+1)} \text{ であるから} \\
& (\text{与式}) = \left(x - \frac{1}{x} \right) \div \left(\frac{2}{x+1} - \frac{1}{x} \right) = \frac{x^2-1}{x} \div \frac{x-1}{x(x+1)} \\
&= \frac{(x+1)(x-1)}{x} \times \frac{x(x+1)}{x-1} = (x+1)^2
\end{aligned}$$

別解 分母・分子に $x(x+1)$ を掛けると

$$(\text{与式}) = \frac{(x^2-1)(x+1)}{2x-(x+1)} = \frac{(x+1)^2(x-1)}{x-1} = (x+1)^2$$

$$(2) \quad (\text{与式}) = \frac{1 - \frac{2}{2+a}}{\left(1 - \frac{2}{2+a} \right) - 1} = \frac{(2+a)-2}{(2+a-2)-(2+a)} = -\frac{a}{2}$$

別解 $\frac{1}{1 - \frac{1}{1 - \frac{2}{2+a}}} = \frac{1}{1 - \frac{1}{\frac{a}{2+a}}} = \frac{1}{1 - \frac{a+2}{a}} = \frac{1}{-\frac{2}{a}} = -\frac{a}{2}$

16 次の式を簡単にせよ。

$$(1) \quad \frac{x-1 + \frac{2}{x+2}}{x+1 - \frac{2}{x+2}} \qquad (2) \quad \frac{\frac{1}{1-x} + \frac{1}{1+x}}{\frac{1}{1-x} - \frac{1}{1+x}} \qquad (3) \quad \frac{1}{1 + \frac{1}{1+x}}$$

解答 (1) $\frac{x+1}{x+3}$ (2) $\frac{1}{x}$ (3) $\frac{x+2}{2x+3}$

解説

$$\begin{aligned}
(1) \quad & (\text{分子}) = \frac{(x-1)(x+2)+2}{x+2} = \frac{x^2+x}{x+2} = \frac{x(x+1)}{x+2} \\
& (\text{分母}) = \frac{(x+1)(x+2)-2}{x+2} = \frac{x^2+3x}{x+2} = \frac{x(x+3)}{x+2}
\end{aligned}$$

$$\text{よって } (\text{与式}) = \frac{x(x+1)}{x+2} \div \frac{x(x+3)}{x+2} = \frac{x(x+1)}{x+2} \times \frac{x+2}{x(x+3)} = \frac{x+1}{x+3}$$

別解 分母・分子に $x+2$ を掛けると

$$(\text{与式}) = \frac{(x-1)(x+2)+2}{(x+1)(x+2)-2} = \frac{x(x+1)}{x(x+3)} = \frac{x+1}{x+3}$$

$$(2) \quad \begin{aligned} \text{(分子)} &= \frac{1+x+1-x}{(1-x)(1+x)} = \frac{2}{(1-x)(1+x)} \\ \text{(分母)} &= \frac{1+x-(1-x)}{(1-x)(1+x)} = \frac{2x}{(1-x)(1+x)} \\ \text{よって} \quad \text{(与式)} &= \frac{2}{(1-x)(1+x)} \div \frac{2x}{(1-x)(1+x)} \\ &= \frac{2}{(1-x)(1+x)} \times \frac{(1-x)(1+x)}{2x} = \frac{1}{x} \end{aligned}$$

別解 分母・分子に $(1-x)(1+x)$ を掛けると

$$\text{(与式)} = \frac{(1+x)+(1-x)}{(1+x)-(1-x)} = \frac{2}{2x} = \frac{1}{x}$$

$$(3) \quad \text{(与式)} = \frac{1 + \frac{1}{x+1}}{\left(1 + \frac{1}{x+1}\right) + 1} = \frac{(x+1)+1}{2(x+1)+1} = \frac{x+2}{2x+3}$$

$$\text{別解 1.} \quad \frac{1}{1 + \frac{1}{1 + \frac{1}{x+1}}} = \frac{1}{1 + \frac{1}{\frac{x+2}{x+1}}} = \frac{1}{1 + \frac{x+1}{x+2}} = \frac{1}{1 + \frac{x+1}{x+2}} = \frac{2x+3}{x+2}$$

$$\text{別解 2.} \quad \frac{1}{1 + \frac{1}{1 + \frac{1}{x+1}}} = \frac{1}{1 + \frac{x+1}{(x+1)+1}} = \frac{1}{1 + \frac{x+1}{x+2}} = \frac{x+2}{2x+3}$$

17 次の計算をせよ。

$$(1) \quad \frac{1}{x-1} - \frac{1}{x+1} - \frac{2}{x^2+1} - \frac{4}{x^4+1}$$

$$(2) \quad \frac{2}{1+2x} + \frac{2}{1-2x} + \frac{1}{(1+2x)(1+3x)} + \frac{1}{(1-2x)(1-3x)}$$

$$\text{解答} \quad (1) \quad \frac{8}{x^8-1} \quad (2) \quad \frac{6}{(1+3x)(1-3x)}$$

解説

$$(1) \quad \begin{aligned} \text{(与式)} &= \frac{(x+1)-(x-1)}{(x-1)(x+1)} - \frac{2}{x^2+1} - \frac{4}{x^4+1} \\ &= \frac{2}{x^2-1} - \frac{2}{x^2+1} - \frac{4}{x^4+1} \\ &= \frac{2\{(x^2+1)-(x^2-1)\}}{(x^2-1)(x^2+1)} - \frac{4}{x^4+1} \\ &= \frac{4}{x^4-1} - \frac{4}{x^4+1} = \frac{4\{(x^4+1)-(x^4-1)\}}{(x^4-1)(x^4+1)} = \frac{8}{x^8-1} \end{aligned}$$

$$(2) \quad \begin{aligned} \text{(与式)} &= \left\{ \frac{2}{1+2x} + \frac{1}{(1+2x)(1+3x)} \right\} + \left\{ \frac{2}{1-2x} + \frac{1}{(1-2x)(1-3x)} \right\} \\ &= \frac{2(1+3x)+1}{(1+2x)(1+3x)} + \frac{2(1-3x)+1}{(1-2x)(1-3x)} \\ &= \frac{3(1+2x)}{(1+2x)(1+3x)} + \frac{3(1-2x)}{(1-2x)(1-3x)} = \frac{3}{1+3x} + \frac{3}{1-3x} \\ &= \frac{3\{(1-3x)+(1+3x)\}}{(1+3x)(1-3x)} = \frac{6}{(1+3x)(1-3x)} \end{aligned}$$

18 次の分数式を約分して簡単にせよ。

$$(1) \quad \frac{15a^2b^2}{40a^3b} \quad (2) \quad \frac{4a^3+8ab^2}{5a^2} \quad (3) \quad \frac{x^2-3x+2}{x^2-4x+3}$$

$$(4) \quad \frac{x^2-2x-3}{x^2-x-2} \quad (5) \quad \frac{a^2-(b-c)^2}{(a+b)^2-c^2} \quad (6) \quad \frac{a^3-a^2b+ab^2}{a^3+b^3}$$

$$\text{解答} \quad (1) \quad \frac{3b}{8a} \quad (2) \quad \frac{4a^2+8b^2}{5a} \quad (3) \quad \frac{x-2}{x-3} \quad (4) \quad \frac{x-3}{x-2} \quad (5) \quad \frac{a-b+c}{a+b+c}$$

$$(6) \quad \frac{a}{a+b}$$

解説

$$(1) \quad \text{与式} = \frac{5a^2b \cdot 3b}{5a^2b \cdot 8a} = \frac{3b}{8a}$$

$$(2) \quad \text{与式} = \frac{a(4a^2+8b^2)}{a \cdot 5a} = \frac{4a^2+8b^2}{5a}$$

$$(3) \quad \text{与式} = \frac{(x-1)(x-2)}{(x-1)(x-3)} = \frac{x-2}{x-3}$$

$$(4) \quad \text{与式} = \frac{(x+1)(x-3)}{(x+1)(x-2)} = \frac{x-3}{x-2}$$

$$(5) \quad \text{与式} = \frac{(a+b-c)(a-b+c)}{(a+b+c)(a+b-c)} = \frac{a-b+c}{a+b+c}$$

$$(6) \quad \text{与式} = \frac{a(a^2-ab+b^2)}{(a+b)(a^2-ab+b^2)} = \frac{a}{a+b}$$

19 次の式を計算せよ。

$$(1) \quad \frac{(7a^2b)^2}{21x^3y^3} \times \frac{3x^2y}{35(ab^2)^2} \quad (2) \quad \frac{3axy^3}{5b^2} \div \frac{6ay^3}{10b^2x}$$

$$(3) \quad \frac{a^2-11a+24}{a^2-6a-16} \times \frac{a^2+2a}{a^2-6a+9} \quad (4) \quad \frac{x^2-8x-20}{3x^2+5x-2} \times \frac{3x^2-31x+10}{x^3-2x^2-80x}$$

$$(5) \quad \frac{a^2+3a+2}{a^2-5a+6} \div \frac{a^2+4a+3}{a^2+a-12} \quad (6) \quad \frac{x^2-9}{x+2} \div (x^2-x-6)$$

$$(7) \quad \frac{6x^2-7x-20}{x^2-4} \times \frac{x^2-x-2}{6x^2-15x} \div \frac{3x^2+7x+4}{x^2+2x}$$

$$\text{解答} \quad (1) \quad \frac{a^2}{5b^2xy^2} \quad (2) \quad x^2 \quad (3) \quad \frac{a}{a-3} \quad (4) \quad \frac{x-10}{x(x+8)} \quad (5) \quad \frac{(a+2)(a+4)}{(a-2)(a+3)}$$

$$(6) \quad \frac{x+3}{(x+2)^2} \quad (7) \quad \frac{1}{3}$$

解説

$$(1) \quad \text{与式} = \frac{49a^4b^2}{21x^3y^3} \times \frac{3x^2y}{35a^2b^4} = \frac{a^2}{5b^2xy^2}$$

$$(2) \quad \text{与式} = \frac{3axy^3}{5b^2} \times \frac{10b^2x}{6ay^3} = x^2$$

$$(3) \quad \text{与式} = \frac{(a-3)(a-8)}{(a+2)(a-8)} \times \frac{a(a+2)}{(a-3)^2} = \frac{a}{a-3}$$

$$(4) \quad \text{与式} = \frac{(x+2)(x-10)}{(3x-1)(x+2)} \times \frac{(3x-1)(x-10)}{x(x+8)(x-10)} = \frac{x-10}{x(x+8)}$$

$$(5) \quad \text{与式} = \frac{(a+1)(a+2)}{(a-2)(a-3)} \times \frac{(a-3)(a+4)}{(a+1)(a+3)} = \frac{(a+2)(a+4)}{(a-2)(a+3)}$$

$$(6) \quad \text{与式} = \frac{(x+3)(x-3)}{x+2} \times \frac{1}{(x+2)(x-3)} = \frac{x+3}{(x+2)^2}$$

$$(7) \quad \text{与式} = \frac{(2x-5)(3x+4)}{(x+2)(x-2)} \times \frac{(x+1)(x-2)}{3x(2x-5)} \times \frac{x(x+2)}{(x+1)(3x+4)} = \frac{1}{3}$$

20 次の式を計算せよ。

$$(1) \quad \frac{x^2+4}{x-2} - \frac{4x}{x-2} \quad (2) \quad \frac{3}{x(3-x)} + \frac{x}{3(x-3)}$$

$$(3) \quad \frac{a}{a+b} + \frac{b}{a-b} \quad (4) \quad \frac{1}{x^2-x} + \frac{1}{x^2-3x+2}$$

$$(5) \quad \frac{2x-1}{x^2-x-6} - \frac{2x+1}{x^2+x-12} \quad (6) \quad \frac{x-2}{2x^2-5x+3} + \frac{3x-1}{2x^2+x-6} + \frac{2x-5}{x^2+x-2}$$

$$\text{解答} \quad (1) \quad x-2 \quad (2) \quad \frac{x+3}{3x} \quad (3) \quad \frac{a^2+b^2}{(a+b)(a-b)} \quad (4) \quad \frac{2}{x(x-2)}$$

$$(5) \quad \frac{2}{(x+2)(x+4)} \quad (6) \quad \frac{4}{x+2}$$

解説

$$(1) \quad \text{与式} = \frac{x^2+4-4x}{x-2} = \frac{(x-2)^2}{x-2} = x-2$$

$$(2) \quad \text{与式} = \frac{3(-3)+x^2}{3x(x-3)} = \frac{(x+3)(x-3)}{3x(x-3)} = \frac{x+3}{3x}$$

$$(3) \quad \text{与式} = \frac{a(a-b)+b(a+b)}{(a+b)(a-b)} = \frac{(a^2-ab)+(ab+b^2)}{(a+b)(a-b)} = \frac{a^2+b^2}{(a+b)(a-b)}$$

$$(4) \quad \text{与式} = \frac{1}{x(x-1)} + \frac{1}{(x-1)(x-2)} = \frac{(x-2)+x}{x(x-1)(x-2)} = \frac{2(x-1)}{x(x-1)(x-2)} = \frac{2}{x(x-2)}$$

$$(5) \quad \begin{aligned} \text{与式} &= \frac{2x-1}{(x+2)(x-3)} - \frac{2x+1}{(x+4)(x-3)} = \frac{(2x-1)(x+4)-(2x+1)(x+2)}{(x+2)(x+4)(x-3)} \\ &= \frac{(2x^2+7x-4)-(2x^2+5x+2)}{(x+2)(x+4)(x-3)} = \frac{2(x-3)}{(x+2)(x+4)(x-3)} = \frac{2}{(x+2)(x+4)} \end{aligned}$$

$$(6) \quad \text{与式} = \frac{x-2}{(x-1)(2x-3)} + \frac{3x-1}{(x+2)(2x-3)} + \frac{2x-5}{(x+2)(x-1)}$$

分母 $= (x+2)(x-1)(2x-3)$ とすると

$$\begin{aligned} \text{分子} &= (x-2)(x+2) + (3x-1)(x-1) + (2x-5)(2x-3) \\ &= (x^2-4) + (3x^2-4x+1) + (4x^2-16x+15) = 8x^2-20x+12 \\ &= 4(x-1)(2x-3) \end{aligned}$$

$$\text{ゆえに} \quad \text{与式} = \frac{4(x-1)(2x-3)}{(x+2)(x-1)(2x-3)} = \frac{4}{x+2}$$

21 次の式を計算せよ。

$$(1) \quad \frac{2}{1+a} + \frac{4}{1+a^2} + \frac{2}{1-a} + \frac{8}{1+a^4}$$

$$(2) \quad \frac{ca}{(a-b)(b-c)} + \frac{ab}{(b-c)(c-a)} + \frac{bc}{(c-a)(a-b)}$$

$$\text{解答} \quad (1) \quad \frac{16}{1-a^8} \quad (2) \quad -1$$

解説

$$(1) \quad \begin{aligned} \text{与式} &= \left(\frac{2}{1+a} + \frac{2}{1-a} \right) + \frac{4}{1+a^2} + \frac{8}{1+a^4} = \frac{2(1-a)+2(1+a)}{(1+a)(1-a)} + \frac{4}{1+a^2} + \frac{8}{1+a^4} \\ &= \left(\frac{4}{1-a^2} + \frac{4}{1+a^2} \right) + \frac{8}{1+a^4} = \frac{4(1+a^2)+4(1-a^2)}{(1-a^2)(1+a^2)} + \frac{8}{1+a^4} \\ &= \frac{8}{1-a^4} + \frac{8}{1+a^4} = \frac{16}{1-a^8} \end{aligned}$$

$$(2) \quad \begin{aligned} \text{与式} &= \frac{ca(c-a)+ab(a-b)+bc(b-c)}{(a-b)(b-c)(c-a)} \\ \text{分子} &= c^2a-ca^2+a^2b-ab^2+bc(b-c) = (b-c)a^2-(b^2-c^2)a+bc(b-c) \\ &= (b-c)\{a^2-(b+c)a+bc\} = (b-c)(a-b)(a-c) \\ &= -(a-b)(b-c)(c-a) \end{aligned}$$

$$\text{よって} \quad \text{与式} = \frac{-(a-b)(b-c)(c-a)}{(a-b)(b-c)(c-a)} = -1$$

22 次の式を計算せよ。

$$(1) \quad \frac{x+2}{x} - \frac{x+3}{x+1} - \frac{x-5}{x-3} + \frac{x-6}{x-4}$$

(2) $\frac{2}{(a-1)(a+1)} + \frac{2}{(a+1)(a+3)} + \frac{2}{(a+3)(a+5)}$

解答 (1) $-\frac{8(2x-3)}{x(x+1)(x-3)(x-4)}$ (2) $\frac{6}{(a-1)(a+5)}$

解説

(1) 与式 $=\left(1+\frac{2}{x}\right)-\left(1+\frac{2}{x+1}\right)-\left(1-\frac{2}{x-3}\right)+\left(1-\frac{2}{x-4}\right)$
 $=\left(\frac{2}{x}-\frac{2}{x+1}\right)+\left(\frac{2}{x-3}-\frac{2}{x-4}\right)=\frac{2}{x(x+1)}-\frac{2}{(x-3)(x-4)}$
 $=-\frac{8(2x-3)}{x(x+1)(x-3)(x-4)}$

(2) 与式 $=\left(\frac{1}{a-1}-\frac{1}{a+1}\right)+\left(\frac{1}{a+1}-\frac{1}{a+3}\right)+\left(\frac{1}{a+3}-\frac{1}{a+5}\right)$
 $=\frac{1}{a-1}-\frac{1}{a+5}=\frac{6}{(a-1)(a+5)}$

23 次の式を計算せよ。

(1) $\frac{x+1}{x}-\frac{x+2}{x+1}-\frac{x-4}{x-3}+\frac{x-5}{x-4}$ (2) $\frac{1}{(x-1)x}+\frac{1}{x(x+1)}+\frac{1}{(x+1)(x+2)}$

解答 (1) $-\frac{4(2x-3)}{x(x+1)(x-3)(x-4)}$ (2) $\frac{3}{(x-1)(x+2)}$

解説

(1) 与式 $=\left(1+\frac{1}{x}\right)-\left(1+\frac{1}{x+1}\right)-\left(1-\frac{1}{x-3}\right)+\left(1-\frac{1}{x-4}\right)$
 $=\left(\frac{1}{x}-\frac{1}{x+1}\right)+\left(\frac{1}{x-3}-\frac{1}{x-4}\right)$
 $=\frac{1}{x(x+1)}-\frac{1}{(x-3)(x-4)}=-\frac{4(2x-3)}{x(x+1)(x-3)(x-4)}$

(2) 与式 $=\left(\frac{1}{x-1}-\frac{1}{x}\right)+\left(\frac{1}{x}-\frac{1}{x+1}\right)+\left(\frac{1}{x+1}-\frac{1}{x+2}\right)$
 $=\frac{1}{x-1}-\frac{1}{x+2}=\frac{3}{(x-1)(x+2)}$