

1 次の式を展開せよ。

- (1) $(x+1)^3$
- (2) $(x-2)^3$
- (3) $(3a+b)^3$
- (4) $(2x-3y)^3$

【解答】 (1) x^3+3x^2+3x+1 (2) $x^3-6x^2+12x-8$ (3) $27a^3+27a^2b+9ab^2+b^3$
(4) $8x^3-36x^2y+54xy^2-27y^3$

解説

- (1) $(x+1)^3=x^3+3x^2\cdot 1+3x\cdot 1^2+1^3=x^3+3x^2+3x+1$
- (2) $(x-2)^3=x^3-3x^2\cdot 2+3x\cdot 2^2-2^3=x^3-6x^2+12x-8$
- (3) $(3a+b)^3=(3a)^3+3\cdot (3a)^2\cdot b+3\cdot 3a\cdot b^2+b^3$
 $=27a^3+27a^2b+9ab^2+b^3$
- (4) $(2x-3y)^3=(2x)^3-3\cdot (2x)^2\cdot 3y+3\cdot 2x\cdot (3y)^2-(3y)^3$
 $=8x^3-36x^2y+54xy^2-27y^3$

2 次の式を展開せよ。

- (1) $(x+2)(x^2-2x+4)$
- (2) $(x-3)(x^2+3x+9)$
- (3) $(3x+y)(9x^2-3xy+y^2)$
- (4) $(2a-3b)(4a^2+6ab+9b^2)$

【解答】 (1) x^3+8 (2) x^3-27 (3) $27x^3+y^3$ (4) $8a^3-27b^3$

解説

- (1) $(x+2)(x^2-2x+4)=(x+2)(x^2-x\cdot 2+2^2)=x^3+2^3=x^3+8$
- (2) $(x-3)(x^2+3x+9)=(x-3)(x^2+x\cdot 3+3^2)=x^3-3^3=x^3-27$
- (3) $(3x+y)(9x^2-3xy+y^2)=(3x+y)\{(3x)^2-3x\cdot y+y^2\}=(3x)^3+y^3=27x^3+y^3$
- (4) $(2a-3b)(4a^2+6ab+9b^2)=(2a-3b)\{(2a)^2+2a\cdot 3b+(3b)^2\}$
 $=(2a)^3-(3b)^3=8a^3-27b^3$

3 次の式を展開せよ。

- (1) $(2xy+1)^3$
- (2) $(a-b)^2(a^2+ab+b^2)^2$

【解答】 (1) $8x^3y^3+12x^2y^2+6xy+1$ (2) $a^6-2a^3b^3+b^6$

解説

- (1) $(2xy+1)^3=(2xy)^3+3\cdot (2xy)^2\cdot 1+3\cdot 2xy\cdot 1^2+1^3$
 $=8x^3y^3+12x^2y^2+6xy+1$
- (2) $(a-b)^2(a^2+ab+b^2)^2=\{(a-b)(a^2+ab+b^2)\}^2=(a^3-b^3)^2$
 $=a^6-2a^3b^3+b^6$

4 次の式を展開せよ。[各 5 点]

- (1) $(2x+1)^3$
- (2) $(a-3b)^3$
- (3) $\left(x+\frac{1}{x}\right)^3$
- (4) $(a-2)(a^2+2a+4)$
- (5) $(2x+y)(4x^2-2xy+y^2)$

【解答】 (1) 与式 $= (2x)^3+3(2x)^2\cdot 1+3\cdot 2x\cdot 1^2+1^3$

$$=8x^3+12x^2+6x+1$$

(2) 与式 $= a^3-3a^2\cdot 3b+3a(3b)^2-(3b)^3$

$$=a^3-9a^2b+27ab^2-27b^3$$

(3) 与式 $= x^3+3x^2\cdot \frac{1}{x}+3x\cdot \frac{1}{x^2}+\frac{1}{x^3}$

$$=x^3+3x+\frac{3}{x}+\frac{1}{x^3}$$

(4) 与式 $= (a-2)(a^2+a\cdot 2+2^2)$

$$=a^3-2^3=a^3-8$$

(5) 与式 $= (2x+y)\{(2x)^2-2x\cdot y+y^2\}$

$$=(2x)^3+y^3=8x^3+y^3$$

解説

- (1) 与式 $= (2x)^3+3(2x)^2\cdot 1+3\cdot 2x\cdot 1^2+1^3$
 $=8x^3+12x^2+6x+1$
- (2) 与式 $= a^3-3a^2\cdot 3b+3a(3b)^2-(3b)^3$
 $=a^3-9a^2b+27ab^2-27b^3$
- (3) 与式 $= x^3+3x^2\cdot \frac{1}{x}+3x\cdot \frac{1}{x^2}+\frac{1}{x^3}$
 $=x^3+3x+\frac{3}{x}+\frac{1}{x^3}$
- (4) 与式 $= (a-2)(a^2+a\cdot 2+2^2)$
 $=a^3-2^3=a^3-8$
- (5) 与式 $= (2x+y)\{(2x)^2-2x\cdot y+y^2\}$
 $=(2x)^3+y^3=8x^3+y^3$

5 次の式を展開せよ。

- (1) $(3x-1)^3$
- (2) $(3x^2-a)(9x^4+3ax^2+a^2)$
- (3) $(x-1)(x+1)(x^2+x+1)(x^2-x+1)$
- (4) $(x+2)(x+4)(x-3)(x-5)$
- (5) $(x+1)^3(x-1)^3$

【解答】 (1) $27x^3-27x^2+9x-1$ (2) $27x^6-a^3$ (3) x^6-1

(4) $x^4-2x^3-25x^2+26x+120$ (5) $x^6-3x^4+3x^2-1$

解説

- (1) $(3x-1)^3=(3x)^3-3(3x)^2\cdot 1+3\cdot 3x\cdot 1^2-1^3$
 $=27x^3-27x^2+9x-1$
- (2) $(3x^2-a)(9x^4+3ax^2+a^2)=(3x^2-a)\{(3x^2)^2+3x^2\cdot a+a^2\}$
 $=(3x^2)^3-a^3$
 $=27x^6-a^3$
- (3) $(x-1)(x+1)(x^2+x+1)(x^2-x+1)$
 $=(x-1)(x^2+x+1)\times (x+1)(x^2-x+1)$
 $=(x^3-1)(x^3+1)=(x^3)^2-1^2$
 $=x^6-1$
- (4) $(x+2)(x+4)(x-3)(x-5)$
 $=(x+2)(x-3)\times (x+4)(x-5)$
 $=(x^2-x-6)(x^2-x-20)$

ここで、 $x^2-x=t$ とおくと

$$(\text{与式})=(t-6)(t-20)=t^2-26t+120$$

$$=(x^2-x)^2-26(x^2-x)+120$$

$$=x^4-2x^3+x^2-26x^2+26x+120$$

$$=x^4-2x^3-25x^2+26x+120$$

- (5) $(x+1)^3(x-1)^3=\{(x+1)(x-1)\}^3=(x^2-1)^3$
 $=(x^2)^3-3(x^2)^2\cdot 1+3x^2\cdot 1^2-1^3$
 $=x^6-3x^4+3x^2-1$

6 (1) の式を展開せよ。また、(2)～(5) の式を因数分解せよ。

- (1) $(x+y+z)^3$
- (2) $27x^3+64$
- (3) x^3-125y^3

- (4) $8x^3-12x^2+6x-1$
- (5) $x^3-y^3-z^3-3xyz$

【解答】 (1) $x^3+y^3+z^3+3x^2y+3xy^2+3y^2z+3yz^2+3z^2x+3zx^2+6xyz$

(2) $(3x+4)(9x^2-12x+16)$ (3) $(x-5y)(x^2+5xy+25y^2)$

(4) $(2x-1)^3$ (5) $(x-y-z)(x^2+y^2+z^2+xy-yz+zx)$

解説

- (1) $(x+y+z)^3=\{(x+y)+z\}^3=(x+y)^3+3(x+y)^2z+3(x+y)z^2+z^3$
 $=x^3+3x^2y+3xy^2+y^3+3(x^2+2xy+y^2)z+3xz^2+3yz^2+z^3$
 $=x^3+3x^2y+3xy^2+y^3+3zx^2+6xyz+3y^2z+3z^2x+3yz^2+z^3$
 $=x^3+y^3+z^3+3x^2y+3xy^2+3y^2z+3yz^2+3z^2x+3zx^2+6xyz$
- (2) $27x^3+64=(3x)^3+4^3=(3x+4)\{(3x)^2-3x\cdot 4+4^2\}=(3x+4)(9x^2-12x+16)$
- (3) $x^3-125y^3=x^3-(5y)^3=(x-5y)\{x^2+x\cdot 5y+(5y)^2\}=(x-5y)(x^2+5xy+25y^2)$
- (4) $8x^3-12x^2+6x-1=(8x^3-1)-(12x^2-6x)=(2x-1)(4x^2+2x+1)-6x(2x-1)$
 $=(2x-1)(4x^2+2x+1-6x)=(2x-1)(4x^2-4x+1)=(2x-1)^3$

別解 $8x^3-12x^2+6x-1=(2x)^3-3\cdot (2x)^2\cdot 1+3\cdot 2x\cdot 1-1^3=(2x-1)^3$

- (5) $x^3-y^3-z^3-3xyz=x^3+(-y)^3+(-z)^3-3x(-y)(-z)$
 $=\{x+(-y)+(-z)\}\{x^2+(-y)^2+(-z)^2-x(-y)-(-y)(-z)-(-z)x\}$
 $=(x-y-z)(x^2+y^2+z^2+xy-yz+zx)$

7 次の式を展開せよ。

- (1) $(t+2)^3(t-2)^3$
- (2) $(a+b)^2(a-b)^2(a^4+a^2b^2+b^4)^2$

【解答】 (1) $t^6-12t^4+48t^2-64$ (2) $a^{12}-2a^6b^6+b^{12}$

解説

- (1) (与式) $=\{(t+2)(t-2)\}^3=(t^2-4)^3$
 $=(t^2)^3-3\cdot (t^2)^2\cdot 4+3\cdot t^2\cdot 4^2-4^3$
 $=t^6-12t^4+48t^2-64$
- (2) (与式) $=\{(a+b)(a-b)(a^4+a^2b^2+b^4)\}^2=\{(a^2-b^2)(a^4+a^2b^2+b^4)\}^2$
 $=\{(a^2)^3-(b^2)^3\}^2=(a^6-b^6)^2=a^{12}-2a^6b^6+b^{12}$

8 次の式を展開せよ。

- (1) $(x+4)^3$
- (2) $(3x-y)^3$
- (3) $(x+3)(x^2-3x+9)$
- (4) $(2x-y)(4x^2+2xy+y^2)$

【解答】 (1) $x^3+12x^2+48x+64$ (2) $27x^3-27x^2y+9xy^2-y^3$ (3) x^3+27

(4) $8x^3-y^3$

解説

- (1) $(x+4)^3=x^3+3\cdot x^2\cdot 4+3\cdot x\cdot 4^2+4^3=x^3+12x^2+48x+64$
- (2) $(3x-y)^3=(3x)^3-3\cdot (3x)^2\cdot y+3\cdot 3x\cdot y^2-y^3=27x^3-27x^2y+9xy^2-y^3$
- (3) $(x+3)(x^2-3x+9)=(x+3)(x^2-x\cdot 3+3^2)=x^3+3^3=x^3+27$
- (4) $(2x-y)(4x^2+2xy+y^2)=(2x-y)\{(2x)^2+(2x)\cdot y+y^2\}=(2x)^3-y^3=8x^3-y^3$

9 次の式を展開せよ。

- (1) $(x+3)^3$
- (2) $(x-2)^3$
- (3) $(x+2y)^3$
- (4) $(x-2)(x^2+2x+4)$

【解答】 (1) $x^3+9x^2+27x+27$ (2) $x^3-6x^2+12x-8$

(3) $x^3+6x^2y+12xy^2+8y^3$ (4) x^3-8

解説

$$(1) \quad (x+3)^3 = x^3 + 3 \cdot x^2 \cdot 3 + 3 \cdot x \cdot 3^2 + 3^3 = x^3 + 9x^2 + 27x + 27$$

$$(2) \quad (x-2)^3 = x^3 - 3 \cdot x^2 \cdot 2 + 3 \cdot x \cdot 2^2 - 2^3 = x^3 - 6x^2 + 12x - 8$$

$$(3) \quad (x+2y)^3 = x^3 + 3 \cdot x^2 \cdot 2y + 3 \cdot x \cdot (2y)^2 + (2y)^3 = x^3 + 6x^2y + 12xy^2 + 8y^3$$

$$(4) \quad (x-2)(x^2+2x+4) = (x-2)(x^2+x \cdot 2+2^2) = x^3-2^3 = x^3-8$$

10 適当な公式を用いて，次の式を展開せよ。

$$(1) \quad (a+3)^3 \qquad (2) \quad (2x-1)^3 \qquad (3) \quad (4x+y)^3$$

$$(4) \quad (-5a+2b)^3 \qquad (5) \quad (ab-2)^3 \qquad (6) \quad (2x^2+3y)^3$$

解答 (1) $a^3+9a^2+27a+27$ (2) $8x^3-12x^2+6x-1$
 (3) $64x^3+48x^2y+12xy^2+y^3$ (4) $-125a^3+150a^2b-60ab^2+8b^3$
 (5) $a^3b^3-6a^2b^2+12ab-8$ (6) $8x^6+36x^4y+54x^2y^2+27y^3$

解説

$$(1) \quad \text{与式} = a^3 + 3 \cdot a^2 \cdot 3 + 3 \cdot a \cdot 3^2 + 3^3 = a^3 + 9a^2 + 27a + 27$$

$$(2) \quad \text{与式} = (2x)^3 - 3 \cdot (2x)^2 \cdot 1 + 3 \cdot 2x \cdot 1^2 - 1^3 = 8x^3 - 12x^2 + 6x - 1$$

$$(3) \quad \text{与式} = (4x)^3 + 3 \cdot (4x)^2 \cdot y + 3 \cdot 4x \cdot y^2 + y^3 = 64x^3 + 48x^2y + 12xy^2 + y^3$$

$$(4) \quad \text{与式} = (-5a)^3 + 3 \cdot (-5a)^2 \cdot 2b + 3 \cdot (-5a) \cdot (2b)^2 + (2b)^3 \\ = -125a^3 + 150a^2b - 60ab^2 + 8b^3$$

$$(5) \quad \text{与式} = (ab)^3 - 3 \cdot (ab)^2 \cdot 2 + 3 \cdot ab \cdot 2^2 - 2^3 = a^3b^3 - 6a^2b^2 + 12ab - 8$$

$$(6) \quad \text{与式} = (2x^2)^3 + 3 \cdot (2x^2)^2 \cdot 3y + 3 \cdot 2x^2 \cdot (3y)^2 + (3y)^3 \\ = 8x^6 + 36x^4y + 54x^2y^2 + 27y^3$$

11 適当な公式を用いて，次の式を展開せよ。

$$(1) \quad (x+4)(x^2-4x+16) \qquad (2) \quad (1-a)(1+a+a^2)$$

$$(3) \quad (2a+3b)(4a^2-6ab+9b^2) \qquad (4) \quad (5x-2y)(25x^2+10xy+4y^2)$$

解答 (1) x^3+64 (2) $1-a^3$ (3) $8a^3+27b^3$ (4) $125x^3-8y^3$

解説

$$(1) \quad \text{与式} = x^3 + 4^3 = x^3 + 64$$

$$(2) \quad \text{与式} = 1^3 - a^3 = 1 - a^3$$

$$(3) \quad \text{与式} = (2a)^3 + (3b)^3 = 8a^3 + 27b^3$$

$$(4) \quad \text{与式} = (5x)^3 - (2y)^3 = 125x^3 - 8y^3$$

12 次の式を展開せよ。

$$(1) \quad (2x+1)^3(2x-1)^3 \qquad (2) \quad (a+2b)^2(a^2-2ab+4b^2)^2$$

$$(3) \quad (x-1)(x+1)(x^2+x+1)(x^2-x+1)$$

解答 (1) $64x^6-48x^4+12x^2-1$ (2) $a^6+16a^3b^3+64b^6$ (3) x^6-1

解説

$$(1) \quad \text{与式} = \{(2x+1)(2x-1)\}^3 = (4x^2-1)^3 = (4x^2)^3 - 3 \cdot (4x^2)^2 \cdot 1 + 3 \cdot 4x^2 \cdot 1^2 - 1^3 \\ = 64x^6 - 48x^4 + 12x^2 - 1$$

$$(2) \quad \text{与式} = \{(a+2b)(a^2-2ab+4b^2)\}^2 = \{a^3+(2b)^3\}^2 = (a^3+8b^3)^2 = a^6+16a^3b^3+64b^6$$

$$(3) \quad \text{与式} = (x-1)(x^2+x+1) \times (x+1)(x^2-x+1) = (x^3-1^3)(x^3+1^3) = (x^3-1)(x^3+1) \\ = x^6-1$$

13 次の式を因数分解せよ。

$$(1) \quad x^3+8 \qquad (2) \quad x^3-64 \qquad (3) \quad a^3+27b^3 \qquad (4) \quad 8x^3-125y^3$$

解答 (1) $(x+2)(x^2-2x+4)$ (2) $(x-4)(x^2+4x+16)$
 (3) $(a+3b)(a^2-3ab+9b^2)$ (4) $(2x-5y)(4x^2+10xy+25y^2)$

解説

$$(1) \quad x^3+8 = x^3+2^3 = (x+2)(x^2-2x+4)$$

$$(2) \quad x^3-64 = x^3-4^3 = (x-4)(x^2+4x+16)$$

$$(3) \quad a^3+27b^3 = a^3+(3b)^3 = (a+3b)(a^2-3ab+9b^2)$$

$$(4) \quad 8x^3-125y^3 = (2x)^3-(5y)^3 = (2x-5y)(4x^2+10xy+25y^2)$$

14 次の式を因数分解せよ。

$$(1) \quad x^6-y^6 \qquad (2) \quad a^6-7a^3-8$$

解答 (1) $(x+y)(x-y)(x^2-xy+y^2)(x^2+xy+y^2)$

$$(2) \quad (a+1)(a-2)(a^2-a+1)(a^2+2a+4)$$

解説

$$(1) \quad x^6-y^6 = (x^3+y^3)(x^3-y^3) \\ = (x+y)(x^2-xy+y^2)(x-y)(x^2+xy+y^2) \\ = (x+y)(x-y)(x^2-xy+y^2)(x^2+xy+y^2)$$

$$(2) \quad a^6-7a^3-8 = (a^3+1)(a^3-8) \\ = (a+1)(a^2-a+1)(a-2)(a^2+2a+4) \\ = (a+1)(a-2)(a^2-a+1)(a^2+2a+4)$$

15 次の式を因数分解せよ。

$$(1) \quad x^6-1 \qquad (2) \quad a^6+9a^3b^3+8b^6$$

解答 (1) $(x+1)(x-1)(x^2-x+1)(x^2+x+1)$

$$(2) \quad (a+b)(a+2b)(a^2-ab+b^2)(a^2-2ab+4b^2)$$

解説

$$(1) \quad x^6-1 = (x^3+1)(x^3-1) = (x+1)(x^2-x+1)(x-1)(x^2+x+1) \\ = (x+1)(x-1)(x^2-x+1)(x^2+x+1)$$

$$(2) \quad a^6+9a^3b^3+8b^6 = (a^3+b^3)(a^3+8b^3) \\ = (a+b)(a^2-ab+b^2)(a+2b)(a^2-2ab+4b^2) \\ = (a+b)(a+2b)(a^2-ab+b^2)(a^2-2ab+4b^2)$$

16 次の式を因数分解せよ。

$$(1) \quad x^3+y^3z^3 \qquad (2) \quad (x+y)^3-1$$

$$(3) \quad a^3-9a^2+27a-27$$

解答 (1) $(x+yz)(x^2-xyz+y^2z^2)$ (2) $(x+y-1)(x^2+2xy+y^2+x+y+1)$

$$(3) \quad (a-3)^3$$

解説

$$(1) \quad x^3+y^3z^3 = x^3+(yz)^3 = (x+yz)\{x^2-x \cdot yz+(yz)^2\} \\ = (x+yz)(x^2-xyz+y^2z^2)$$

$$(2) \quad (x+y)^3-1 = (x+y)^3-1^3 = \{(x+y)-1\}\{(x+y)^2+(x+y) \cdot 1+1^2\} \\ = (x+y-1)(x^2+2xy+y^2+x+y+1)$$

$$(3) \quad a^3-9a^2+27a-27 = (a^3-27) + (-9a^2+27a) \\ = (a-3)(a^2+3a+9) - 9a(a-3) \\ = (a-3)\{a^2+3a+9-9a\} \\ = (a-3)(a^2-6a+9) \\ = (a-3)(a-3)^2 \\ = (a-3)^3$$

別解 $a^3-9a^2+27a-27 = a^3-3a^2 \cdot 3+3a \cdot 3^2-3^3 \\ = (a-3)^3$

17 次の式を因数分解せよ。[各 5 点]

$$(1) \quad x^3-27 \qquad (2) \quad 125a^3+1 \qquad (3) \quad 8x^3-27y^3$$

$$(4) \quad 64a^6-b^6 \qquad (5) \quad x^6+7x^3y^3-8y^6$$

解答 (1) 与式 $= x^3-3^3 = (x-3)(x^2+3x+9)$

$$(2) \quad \text{与式} = (5a)^3+1^3 = (5a+1)(25a^2-5a+1)$$

$$(3) \quad \text{与式} = (2x)^3-(3y)^3 = (2x-3y)(4x^2+6xy+9y^2)$$

$$(4) \quad \text{与式} = (8a^3+b^3)(8a^3-b^3) \\ = (2a+b)(4a^2-2ab+b^2)(2a-b)(4a^2+2ab+b^2) \\ = (2a+b)(2a-b)(4a^2-2ab+b^2)(4a^2+2ab+b^2) \\ (5) \quad \text{与式} = (x^3+8y^3)(x^3-y^3) \\ = (x+2y)(x^2-2xy+4y^2)(x-y)(x^2+xy+y^2) \\ = (x+2y)(x-y)(x^2-2xy+4y^2)(x^2+xy+y^2)$$

解説

$$(1) \quad \text{与式} = x^3-3^3 = (x-3)(x^2+3x+9)$$

$$(2) \quad \text{与式} = (5a)^3+1^3 = (5a+1)(25a^2-5a+1)$$

$$(3) \quad \text{与式} = (2x)^3-(3y)^3 = (2x-3y)(4x^2+6xy+9y^2)$$

$$(4) \quad \text{与式} = (8a^3+b^3)(8a^3-b^3) \\ = (2a+b)(4a^2-2ab+b^2)(2a-b)(4a^2+2ab+b^2) \\ = (2a+b)(2a-b)(4a^2-2ab+b^2)(4a^2+2ab+b^2) \\ (5) \quad \text{与式} = (x^3+8y^3)(x^3-y^3) \\ = (x+2y)(x^2-2xy+4y^2)(x-y)(x^2+xy+y^2) \\ = (x+2y)(x-y)(x^2-2xy+4y^2)(x^2+xy+y^2)$$

18 $8x^3+36x^2+54x+27$ を因数分解せよ。[10 点]

解答 与式 $= (8x^3+27) + (36x^2+54x) \\ = (2x+3)(4x^2-6x+9) + 18x(2x+3) \\ = (2x+3)\{(4x^2-6x+9) + 18x\} \\ = (2x+3)(4x^2+12x+9) = (2x+3)^3$

別解 与式 $= (2x)^3+3(2x)^2 \cdot 3+3 \cdot 2x \cdot 3^2+3^3 = (2x+3)^3$

解説

$$\text{与式} = (8x^3+27) + (36x^2+54x) \\ = (2x+3)(4x^2-6x+9) + 18x(2x+3) \\ = (2x+3)\{(4x^2-6x+9) + 18x\} \\ = (2x+3)(4x^2+12x+9) = (2x+3)^3$$

別解 与式 $= (2x)^3+3(2x)^2 \cdot 3+3 \cdot 2x \cdot 3^2+3^3 = (2x+3)^3$

19 次の式を因数分解せよ。

$$(1) \quad 2x^3+16y^3 \qquad (2) \quad (x+1)^3-27$$

解答 (1) $2(x+2y)(x^2-2xy+4y^2)$ (2) $(x-2)(x^2+5x+13)$

解説

$$(1) \quad 2x^3+16y^3 = 2(x^3+8y^3) = 2\{x^3+(2y)^3\} \\ = 2(x+2y)(x^2-2xy+4y^2) \\ (2) \quad (x+1)^3-27 = (x+1)^3-3^3 \\ = \{(x+1)-3\}\{(x+1)^2+3(x+1)+3^2\} \\ = (x-2)\{(x^2+2x+1) + 3x+3+9\} \\ = (x-2)(x^2+5x+13)$$

20 次の式を因数分解せよ。

(1) $x^3 - 27$ (2) $64a^3 + 125b^3$ (3) $x^4 + 8x$

【解答】 (1) $(x-3)(x^2+3x+9)$ (2) $(4a+5b)(16a^2-20ab+25b^2)$
(3) $x(x+2)(x^2-2x+4)$

【解説】

(1) $x^3 - 27 = x^3 - 3^3$
 $= (x-3)(x^2 + x \cdot 3 + 3^2)$
 $= (x-3)(x^2 + 3x + 9)$
(2) $64a^3 + 125b^3 = (4a)^3 + (5b)^3$
 $= (4a+5b)\{(4a)^2 - 4a \cdot 5b + (5b)^2\}$
 $= (4a+5b)(16a^2 - 20ab + 25b^2)$
(3) $x^4 + 8x = x(x^3 + 8) = x(x^3 + 2^3)$
 $= x(x+2)(x^2 - x \cdot 2 + 2^2)$
 $= x(x+2)(x^2 - 2x + 4)$

21 次の式を因数分解せよ。

(1) $8x^3 + 27y^3$ (2) $64x^3 - 1$ (3) $54x^3 + 16$

【解答】 (1) $(2x+3y)(4x^2-6xy+9y^2)$ (2) $(4x-1)(16x^2+4x+1)$
(3) $2(3x+2)(9x^2-6x+4)$

【解説】

(1) $8x^3 + 27y^3 = (2x)^3 + (3y)^3$
 $= (2x+3y)\{(2x)^2 - 2x \cdot 3y + (3y)^2\}$
 $= (2x+3y)(4x^2 - 6xy + 9y^2)$
(2) $64x^3 - 1 = (4x)^3 - 1^3$
 $= (4x-1)\{(4x)^2 + 4x \cdot 1 + 1^2\}$
 $= (4x-1)(16x^2 + 4x + 1)$
(3) $54x^3 + 16 = 2(27x^3 + 8)$
 $= 2\{(3x)^3 + 2^3\}$
 $= 2(3x+2)\{(3x)^2 - 3x \cdot 2 + 2^2\}$
 $= 2(3x+2)(9x^2 - 6x + 4)$

22 次の式を因数分解せよ。

(1) $x^6 - 1$ (2) $(x+y)^6 - (x-y)^6$
(3) $x^6 - 19x^3 - 216$ (4) $x^6 - 2x^3 + 1$

【解答】 (1) $(x+1)(x-1)(x^2-x+1)(x^2+x+1)$ (2) $4xy(x^2+3y^2)(3x^2+y^2)$
(3) $(x+2)(x-3)(x^2-2x+4)(x^2+3x+9)$ (4) $(x-1)^2(x^2+x+1)^2$

【解説】

(1) $x^6 - 1 = (x^3)^2 - 1 = (x^3+1)(x^3-1)$
 $= (x+1)(x^2-x+1)(x-1)(x^2+x+1)$
 $= (x+1)(x-1)(x^2-x+1)(x^2+x+1)$
【別解】 $x^6 - 1 = (x^2)^3 - 1 = (x^2-1)(x^4+x^2+1)$
 $= (x+1)(x-1)(x^4+x^2+1)$
ここで $x^4 + x^2 + 1 = (x^4 + 2x^2 + 1) - x^2$
 $= (x^2+1)^2 - x^2$
 $= (x^2+1+x)(x^2+1-x)$
よって $x^6 - 1 = (x+1)(x-1)(x^2+x+1)(x^2-x+1)$
(2) $(x+y)^6 - (x-y)^6 = \{(x+y)^3\}^2 - \{(x-y)^3\}^2$

$= \{(x+y)^3 + (x-y)^3\}\{(x+y)^3 - (x-y)^3\}$
 $= (x^3 + 3x^2y + 3xy^2 + y^3 + x^3 - 3x^2y + 3xy^2 - y^3)$
 $\times (x^3 + 3x^2y + 3xy^2 + y^3 - x^3 + 3x^2y - 3xy^2 + y^3)$
 $= (2x^3 + 6xy^2)(6x^2y + 2y^3)$
 $= 2x(x^2 + 3y^2) \cdot 2y(3x^2 + y^2)$
 $= 4xy(x^2 + 3y^2)(3x^2 + y^2)$
(3) $x^6 - 19x^3 - 216 = (x^3)^2 - 19x^3 - 216 = (x^3+8)(x^3-27)$
 $= (x+2)(x^2-2x+4)(x-3)(x^2+3x+9)$
 $= (x+2)(x-3)(x^2-2x+4)(x^2+3x+9)$
(4) $x^6 - 2x^3 + 1 = (x^3)^2 - 2x^3 + 1$
 $= (x^3-1)^2$
 $= \{(x-1)(x^2+x+1)\}^2$
 $= (x-1)^2(x^2+x+1)^2$

23 次の式を因数分解せよ。

(1) $3a^3 - 81b^3$ (2) $125x^4 + 8xy^3$
(3) $t^3 - t^2 + \frac{t}{3} - \frac{1}{27}$ (4) $x^3 + 3x^2 - 4x - 12$

【解答】 (1) $3(a-3b)(a^2+3ab+9b^2)$ (2) $x(5x+2y)(25x^2-10xy+4y^2)$
(3) $\left(t-\frac{1}{3}\right)^3$ (4) $(x+3)(x+2)(x-2)$

【解説】

(1) $3a^3 - 81b^3 = 3(a^3 - 27b^3) = 3\{a^3 - (3b)^3\}$
 $= 3(a-3b)\{a^2 + a \cdot 3b + (3b)^2\}$
 $= 3(a-3b)(a^2 + 3ab + 9b^2)$
(2) $125x^4 + 8xy^3 = x(125x^3 + 8y^3) = x\{(5x)^3 + (2y)^3\}$
 $= x(5x+2y)\{(5x)^2 - 5x \cdot 2y + (2y)^2\}$
 $= x(5x+2y)(25x^2 - 10xy + 4y^2)$
(3) $t^3 - t^2 + \frac{t}{3} - \frac{1}{27} = t^3 - 3t^2 \cdot \frac{1}{3} + 3t \cdot \left(\frac{1}{3}\right)^2 - \left(\frac{1}{3}\right)^3$
 $= \left(t - \frac{1}{3}\right)^3$
(4) $x^3 + 3x^2 - 4x - 12 = x^2(x+3) - 4(x+3) = (x+3)(x^2-4)$
 $= (x+3)(x+2)(x-2)$

24 次の式を因数分解せよ。

(1) $x^3 - 5x^2 - 4x + 20$ (2) $8x^3 + 6x^2 + 3x + 1$
(3) $x^2y + 4y^2z - 4y^3 - x^2z$ (4) $a^4 + a^2c - ab^3 + abc + b^2c$

【解答】 (1) $(x+2)(x-2)(x-5)$ (2) $(2x+1)(4x^2+x+1)$
(3) $(x+2y)(x-2y)(y-z)$ (4) $(a^2+ab+b^2)(a^2-ab+c)$

【解説】

(1) 与式 $= x^2(x-5) - 4(x-5) = (x^2-4)(x-5) = (x+2)(x-2)(x-5)$
【別解】 与式 $= (x^3-4x) - (5x^2-20) = x(x^2-4) - 5(x^2-4)$
 $= (x-5)(x^2-4) = (x-5)(x+2)(x-2)$
(2) 与式 $= (8x^3+1) + (6x^2+3x) = (2x+1)(4x^2-2x+1) + 3x(2x+1)$
 $= (2x+1)\{(4x^2-2x+1) + 3x\} = (2x+1)(4x^2+x+1)$
(3) 与式 $= x^2(y-z) - 4y^2(y-z) = (x^2-4y^2)(y-z) = (x+2y)(x-2y)(y-z)$
【参考】 次数の低い z について整理してもよい。
与式 $= (4y^2-x^2)z + x^2y - 4y^3 = (4y^2-x^2)z + y(x^2-4y^2) = (x^2-4y^2)(y-z)$

$= (x+2y)(x-2y)(y-z)$
(4) 与式 $= (a^2+ab+b^2)c + a^4 - ab^3 = (a^2+ab+b^2)c + a(a^3-b^3)$
 $= (a^2+ab+b^2)c + a(a-b)(a^2+ab+b^2) = (a^2+ab+b^2)\{c + a(a-b)\}$
 $= (a^2+ab+b^2)(a^2-ab+c)$

25 適当な公式を用いて、次の式を因数分解せよ。

(1) $8x^3 + 27$ (2) $a^3 - 125b^3$ (3) $128x^3 + 2y^3$ (4) $a^3b^3 - c^3$

【解答】 (1) $(2x+3)(4x^2-6x+9)$ (2) $(a-5b)(a^2+5ab+25b^2)$
(3) $2(4x+y)(16x^2-4xy+y^2)$ (4) $(ab-c)(a^2b^2+abc+c^2)$

【解説】

(1) 与式 $= (2x)^3 + 3^3 = (2x+3)\{(2x)^2 - 2x \cdot 3 + 3^2\} = (2x+3)(4x^2-6x+9)$
(2) 与式 $= a^3 - (5b)^3 = (a-5b)\{a^2 + a \cdot 5b + (5b)^2\} = (a-5b)(a^2+5ab+25b^2)$
(3) 与式 $= 2(64x^3 + y^3) = 2\{(4x)^3 + y^3\} = 2(4x+y)\{(4x)^2 - 4x \cdot y + y^2\}$
 $= 2(4x+y)(16x^2 - 4xy + y^2)$
(4) 与式 $= (ab)^3 - c^3 = (ab-c)\{(ab)^2 + ab \cdot c + c^2\} = (ab-c)(a^2b^2+abc+c^2)$

26 次の式を因数分解せよ。

(1) $64x^6 - 1$ (2) $a^6 + 26a^3b^3 - 27b^6$ (3) $(x-y)^3 + 8$

【解答】 (1) $(2x+1)(2x-1)(4x^2-2x+1)(4x^2+2x+1)$
(2) $(a-b)(a+3b)(a^2+ab+b^2)(a^2-3ab+9b^2)$
(3) $(x-y+2)(x^2-2xy+y^2-2x+2y+4)$

【解説】

(1) 与式 $= (8x^3+1)(8x^3-1) = (2x+1)(4x^2-2x+1)(2x-1)(4x^2+2x+1)$
 $= (2x+1)(2x-1)(4x^2-2x+1)(4x^2+2x+1)$
【別解】 与式 $= (4x^2)^3 - 1^3 = (4x^2-1)(16x^4+4x^2+1) = (4x^2-1)\{(4x^2+1)^2 - 4x^2\}$
 $= (2x+1)(2x-1)(4x^2-2x+1)(4x^2+2x+1)$
(2) 与式 $= (a^3-b^3)(a^3+27b^3) = (a-b)(a^2+ab+b^2)(a+3b)(a^2-3ab+9b^2)$
 $= (a-b)(a+3b)(a^2+ab+b^2)(a^2-3ab+9b^2)$
(3) 与式 $= (x-y)^3 + 2^3 = \{(x-y) + 2\}\{(x-y)^2 - (x-y) \cdot 2 + 2^2\}$
 $= (x-y+2)(x^2-2xy+y^2-2x+2y+4)$

27 次の式を因数分解せよ。

(1) $x^3 - 3x^2 + 6x - 8$ (2) $8a^3 - 36a^2b + 54ab^2 - 27b^3$

【解答】 (1) $(x-2)(x^2-x+4)$ (2) $(2a-3b)^3$

【解説】

(1) 与式 $= x^3 - 8 - (3x^2 - 6x) = (x-2)(x^2+2x+4) - 3x(x-2)$
 $= (x-2)\{(x^2+2x+4) - 3x\} = (x-2)(x^2-x+4)$
(2) 与式 $= (2a)^3 - 3 \cdot (2a)^2 \cdot 3b + 3 \cdot 2a \cdot (3b)^2 - (3b)^3 = (2a-3b)^3$
【別解】 与式 $= 8a^3 - 27b^3 - (36a^2b - 54ab^2) = (2a-3b)(4a^2+6ab+9b^2) - 18ab(2a-3b)$
 $= (2a-3b)\{(4a^2+6ab+9b^2) - 18ab\} = (2a-3b)(4a^2-12ab+9b^2)$
 $= (2a-3b)(2a-3b)^2 = (2a-3b)^3$