1 次の問いに答えよ。

(1) 49 の平方根を求めよ。　(2) $\sqrt{25}$ の値を求めよ。

解答　(1) ± 7　(2) 5

解説

(1) ± 7

(2) $\sqrt{25}=5$

2 $(\sqrt{7})^2$，$(-\sqrt{15})^2$ の値を，それぞれ求めよ。

解答　$(\sqrt{7})^2=7$，$(-\sqrt{15})^2=15$

解説

$(\sqrt{7})^2=7$，　$(-\sqrt{15})^2=15$

3 次の式を計算せよ。

(1) $4\sqrt{3}+5\sqrt{3}-7\sqrt{3}$　(2) $3\sqrt{50}-4\sqrt{18}+\sqrt{32}$

(3) $(\sqrt{7}+2)(\sqrt{7}-2)$　(4) $(4\sqrt{2}-3\sqrt{3})(5\sqrt{2}+2\sqrt{3})$

(5) $(\sqrt{3}+2\sqrt{6})^2$　(6) $(3\sqrt{2}-2\sqrt{7})^2$

解答　(1) $2\sqrt{3}$　(2) $7\sqrt{2}$　(3) 3　(4) $22-7\sqrt{6}$　(5) $27+12\sqrt{2}$

(6) $46-12\sqrt{14}$

解説

(1) $4\sqrt{3}+5\sqrt{3}-7\sqrt{3}=(4+5-7)\sqrt{3}=2\sqrt{3}$

(2) $3\sqrt{50}-4\sqrt{18}+\sqrt{32}=15\sqrt{2}-12\sqrt{2}+4\sqrt{2}$

$=(15-12+4)\sqrt{2}=7\sqrt{2}$

(3) $(\sqrt{7}+2)(\sqrt{7}-2)=(\sqrt{7})^2-2^2=7-4=3$

(4) $(4\sqrt{2}-3\sqrt{3})(5\sqrt{2}+2\sqrt{3})=4\sqrt{2}\cdot5\sqrt{2}+4\sqrt{2}\cdot2\sqrt{3}-3\sqrt{3}\cdot5\sqrt{2}-3\sqrt{3}\cdot2\sqrt{3}$

$=40+8\sqrt{6}-15\sqrt{6}-18$

$=22-7\sqrt{6}$

(5) $(\sqrt{3}+2\sqrt{6})^2=(\sqrt{3})^2+2\cdot\sqrt{3}\cdot2\sqrt{6}+(2\sqrt{6})^2$

$=3+12\sqrt{2}+24=27+12\sqrt{2}$

(6) $(3\sqrt{2}-2\sqrt{7})^2=(3\sqrt{2})^2-2\cdot3\sqrt{2}\cdot2\sqrt{7}+(2\sqrt{7})^2$

$=18-12\sqrt{14}+28=46-12\sqrt{14}$

4 $\dfrac{2\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ の分母を有理化せよ。

解答　$8+3\sqrt{6}$

解説

$$\frac{2\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}=\frac{(2\sqrt{3}+\sqrt{2})(\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})}$$

$$=\frac{6+2\sqrt{6}+\sqrt{6}+2}{(\sqrt{3})^2-(\sqrt{2})^2}=8+3\sqrt{6}$$

5 次の式の分母を有理化せよ。

(1) $\dfrac{18}{\sqrt{6}}$　(2) $\dfrac{\sqrt{3}}{2+\sqrt{3}}$　(3) $\dfrac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$　(4) $\dfrac{3\sqrt{7}-\sqrt{3}}{\sqrt{7}+\sqrt{3}}$

解答　(1) $3\sqrt{6}$　(2) $2\sqrt{3}-3$　(3) $\dfrac{7+2\sqrt{10}}{3}$　(4) $6-\sqrt{21}$

解説

(1) $\dfrac{18}{\sqrt{6}}=\dfrac{18\times\sqrt{6}}{\sqrt{6}\times\sqrt{6}}=\dfrac{18\sqrt{6}}{6}=3\sqrt{6}$

(2) $\dfrac{\sqrt{3}}{2+\sqrt{3}}=\dfrac{\sqrt{3}(2-\sqrt{3})}{(2+\sqrt{3})(2-\sqrt{3})}$

$=\dfrac{2\sqrt{3}-(\sqrt{3})^2}{2^2-(\sqrt{3})^2}$

$=2\sqrt{3}-3$

(3) $\dfrac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}=\dfrac{(\sqrt{5}+\sqrt{2})^2}{(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})}$

$=\dfrac{5+2\sqrt{10}+2}{(\sqrt{5})^2-(\sqrt{2})^2}=\dfrac{7+2\sqrt{10}}{3}$

(4) $\dfrac{3\sqrt{7}-\sqrt{3}}{\sqrt{7}+\sqrt{3}}=\dfrac{(3\sqrt{7}-\sqrt{3})(\sqrt{7}-\sqrt{3})}{(\sqrt{7}+\sqrt{3})(\sqrt{7}-\sqrt{3})}$

$=\dfrac{21-3\sqrt{21}-\sqrt{21}+3}{(\sqrt{7})^2-(\sqrt{3})^2}$

$=\dfrac{24-4\sqrt{21}}{4}$

$=6-\sqrt{21}$

6 次の式を計算せよ。

(1) $2\sqrt{5}+\sqrt{45}-\sqrt{125}$　(2) $\sqrt{48}+\sqrt{32}-\sqrt{27}-\sqrt{50}$

(3) $(2\sqrt{3}-5\sqrt{2})(3\sqrt{2}+\sqrt{3})$　(4) $(2\sqrt{6}-\sqrt{18})(\sqrt{6}+3\sqrt{8})$

(5) $(1+\sqrt{2}+\sqrt{3})^2$　(6) $(2-\sqrt{3}+\sqrt{7})(2-\sqrt{3}-\sqrt{7})$

解答　(1) 0　(2) $\sqrt{3}-\sqrt{2}$　(3) $\sqrt{6}-24$　(4) $18\sqrt{3}-24$

(5) $6+2\sqrt{2}+2\sqrt{3}+2\sqrt{6}$　(6) $-4\sqrt{3}$

解説

(1) 与式$=2\sqrt{5}+3\sqrt{5}-5\sqrt{5}$

$=(2+3-5)\sqrt{5}=0$

(2) 与式$=4\sqrt{3}+4\sqrt{2}-3\sqrt{3}-5\sqrt{2}=\sqrt{3}-\sqrt{2}$

(3) 与式$=(2\sqrt{3}-5\sqrt{2})(\sqrt{3}+3\sqrt{2})$

$=2(\sqrt{3})^2+(6-5)\sqrt{3}\sqrt{2}-15(\sqrt{2})^2$

$=2\cdot3+\sqrt{6}-15\cdot2=\sqrt{6}-24$

(4) 与式$=(2\sqrt{6}-3\sqrt{2})(\sqrt{6}+6\sqrt{2})$

$=2(\sqrt{6})^2+(12-3)\sqrt{6}\sqrt{2}-18(\sqrt{2})^2$

$=2\cdot6+9\sqrt{12}-18\cdot2=18\sqrt{3}-24$

(5) 与式$=\{(1+\sqrt{2})+\sqrt{3}\}^2$

$=(1+\sqrt{2})^2+2(1+\sqrt{2})\sqrt{3}+(\sqrt{3})^2$

$=1+2\sqrt{2}+2+2\sqrt{3}+2\sqrt{6}+3$

$=6+2\sqrt{2}+2\sqrt{3}+2\sqrt{6}$

(6) 与式$=\{(2-\sqrt{3})+\sqrt{7}\}\{(2-\sqrt{3})-\sqrt{7}\}$

$=(2-\sqrt{3})^2-(\sqrt{7})^2$

$=4-4\sqrt{3}+3-7$

$=-4\sqrt{3}$

7 次の式の分母を有理化せよ。

(1) $\dfrac{\sqrt{2}}{\sqrt{3}-\sqrt{5}}$　(2) $\dfrac{2\sqrt{5}+\sqrt{2}}{\sqrt{5}+2\sqrt{2}}$

解答　(1) $-\dfrac{\sqrt{6}+\sqrt{10}}{2}$　(2) $\sqrt{10}-2$

解説

(1) 与式$=\dfrac{\sqrt{2}(\sqrt{3}+\sqrt{5})}{(\sqrt{3}-\sqrt{5})(\sqrt{3}+\sqrt{5})}=\dfrac{\sqrt{6}+\sqrt{10}}{3-5}$

$=-\dfrac{\sqrt{6}+\sqrt{10}}{2}$

(2) 与式$=\dfrac{(\sqrt{2}+2\sqrt{5})(2\sqrt{2}-\sqrt{5})}{(2\sqrt{2}+\sqrt{5})(2\sqrt{2}-\sqrt{5})}$

$=\dfrac{4+3\sqrt{10}-10}{8-5}=\sqrt{10}-2$

8 次の式を計算せよ。

(1) $\dfrac{\sqrt{5}}{\sqrt{3}-1}-\dfrac{\sqrt{3}}{\sqrt{5}+\sqrt{3}}$　(2) $\dfrac{\sqrt{7}+1}{\sqrt{7}-\sqrt{3}}-\dfrac{\sqrt{7}-1}{\sqrt{7}+\sqrt{3}}$

(3) $\dfrac{1}{1+\sqrt{2}}+\dfrac{1}{\sqrt{2}+\sqrt{3}}+\dfrac{1}{\sqrt{3}+2}$　(4) $\dfrac{\sqrt{2}+1}{\sqrt{2}-1}+\dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}+\dfrac{4(1+\sqrt{2})}{1-\sqrt{3}}$

解答　(1) $\dfrac{3+\sqrt{5}}{2}$　(2) $\dfrac{\sqrt{7}+\sqrt{21}}{2}$　(3) 1　(4) $6-2\sqrt{3}$

解説

(1) 与式$=\dfrac{\sqrt{5}(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}-\dfrac{\sqrt{3}(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})}$

$=\dfrac{\sqrt{15}+\sqrt{5}}{2}-\dfrac{\sqrt{15}-3}{2}=\dfrac{3+\sqrt{5}}{2}$

(2) 与式$=\dfrac{(\sqrt{7}+1)(\sqrt{7}+\sqrt{3})}{(\sqrt{7}-\sqrt{3})(\sqrt{7}+\sqrt{3})}-\dfrac{(\sqrt{7}-1)(\sqrt{7}-\sqrt{3})}{(\sqrt{7}+\sqrt{3})(\sqrt{7}-\sqrt{3})}$

$=\dfrac{7+\sqrt{21}+\sqrt{7}+\sqrt{3}}{4}-\dfrac{7-\sqrt{21}-\sqrt{7}+\sqrt{3}}{4}$

$=\dfrac{2\sqrt{7}+2\sqrt{21}}{4}=\dfrac{\sqrt{7}+\sqrt{21}}{2}$

(3) 与式$=\dfrac{1-\sqrt{2}}{(1+\sqrt{2})(1-\sqrt{2})}+\dfrac{\sqrt{2}-\sqrt{3}}{(\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3})}+\dfrac{\sqrt{3}-2}{(\sqrt{3}+2)(\sqrt{3}-2)}$

$=-(1-\sqrt{2})-(\sqrt{2}-\sqrt{3})-(\sqrt{3}-2)=1$

(4) 与式$=\dfrac{(\sqrt{2}+1)^2}{(\sqrt{2}-1)(\sqrt{2}+1)}+\dfrac{(\sqrt{3}+\sqrt{2})^2}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})}+\dfrac{4(1+\sqrt{2})(1+\sqrt{3})}{(1-\sqrt{3})(1+\sqrt{3})}$

$=3+2\sqrt{2}+5+2\sqrt{6}-2(1+\sqrt{2}+\sqrt{3}+\sqrt{6})$

$=6-2\sqrt{3}$

$\boxed{9}$ 次の式を計算せよ。[各 4 点]

(1) $2\sqrt{2}+\sqrt{50}$　(2) $5\sqrt{7}-\sqrt{112}+\sqrt{63}$　(3) $\sqrt{2}(3\sqrt{2}-\sqrt{6})$

(4) $(\sqrt{3}+\sqrt{7})^2$　(5) $(3\sqrt{2}-2\sqrt{3})^2$　(6) $(\sqrt{7}-\sqrt{3})(\sqrt{7}+\sqrt{3})$

(7) $(\sqrt{20}-\sqrt{3})(3\sqrt{5}-\sqrt{12})$　(8) $\dfrac{14}{\sqrt{7}}$　(9) $\dfrac{5+\sqrt{3}}{2-\sqrt{3}}$　(10) $\dfrac{\sqrt{7}-\sqrt{2}}{\sqrt{7}+\sqrt{2}}$

解答 (1) 与式$=2\sqrt{2}+5\sqrt{2}=7\sqrt{2}$

(2) 与式$=5\sqrt{7}-4\sqrt{7}+3\sqrt{7}=4\sqrt{7}$

(3) 与式$=2\cdot3-\sqrt{2^2\cdot3}=6-2\sqrt{3}$

(4) 与式$=3+2\sqrt{21}+7=10+2\sqrt{21}$

(5) 与式$=9\cdot2-12\sqrt{6}+4\cdot3=30-12\sqrt{6}$

(6) 与式$=7-3=4$

(7) 与式$=(2\sqrt{5}-\sqrt{3})(3\sqrt{5}-2\sqrt{3})=6\cdot5-(4+3)\sqrt{15}+2\cdot3$

$=36-7\sqrt{15}$

(8) 与式$=\dfrac{14\sqrt{7}}{\sqrt{7}\sqrt{7}}=\dfrac{14\sqrt{7}}{7}=2\sqrt{7}$

(9) 与式$=\dfrac{(5+\sqrt{3})(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})}=\dfrac{10+7\sqrt{3}+3}{4-3}=13+7\sqrt{3}$

(10) 与式$=\dfrac{(\sqrt{7}-\sqrt{2})^2}{(\sqrt{7}+\sqrt{2})(\sqrt{7}-\sqrt{2})}=\dfrac{7-2\sqrt{14}+2}{7-2}=\dfrac{9-2\sqrt{14}}{5}$

解説

(1) 与式$=2\sqrt{2}+5\sqrt{2}=7\sqrt{2}$

(2) 与式$=5\sqrt{7}-4\sqrt{7}+3\sqrt{7}=4\sqrt{7}$

(3) 与式$=2\cdot3-\sqrt{2^2\cdot3}=6-2\sqrt{3}$

(4) 与式$=3+2\sqrt{21}+7=10+2\sqrt{21}$

(5) 与式$=9\cdot2-12\sqrt{6}+4\cdot3=30-12\sqrt{6}$

(6) 与式$=7-3=4$

(7) 与式$=(2\sqrt{5}-\sqrt{3})(3\sqrt{5}-2\sqrt{3})=6\cdot5-(4+3)\sqrt{15}+2\cdot3$

$=36-7\sqrt{15}$

(8) 与式$=\dfrac{14\sqrt{7}}{\sqrt{7}\sqrt{7}}=\dfrac{14\sqrt{7}}{7}=2\sqrt{7}$

(9) 与式$=\dfrac{(5+\sqrt{3})(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})}=\dfrac{10+7\sqrt{3}+3}{4-3}=13+7\sqrt{3}$

(10) 与式$=\dfrac{(\sqrt{7}-\sqrt{2})^2}{(\sqrt{7}+\sqrt{2})(\sqrt{7}-\sqrt{2})}=\dfrac{7-2\sqrt{14}+2}{7-2}=\dfrac{9-2\sqrt{14}}{5}$

$\boxed{10}$ $\dfrac{1}{\sqrt{2}+\sqrt{3}-\sqrt{5}}-\dfrac{1}{\sqrt{2}+\sqrt{3}+\sqrt{5}}$ を簡単にせよ。[20 点]

解答 与式$=\dfrac{1}{(\sqrt{2}+\sqrt{3})-\sqrt{5}}-\dfrac{1}{(\sqrt{2}+\sqrt{3})+\sqrt{5}}$

$=\dfrac{(\sqrt{2}+\sqrt{3}+\sqrt{5})-(\sqrt{2}+\sqrt{3}-\sqrt{5})}{(\sqrt{2}+\sqrt{3})^2-(\sqrt{5})^2}$

$=\dfrac{2\sqrt{5}}{2\sqrt{6}}=\dfrac{\sqrt{5}}{\sqrt{6}}=\dfrac{\sqrt{30}}{6}$

解説

与式$=\dfrac{1}{(\sqrt{2}+\sqrt{3})-\sqrt{5}}-\dfrac{1}{(\sqrt{2}+\sqrt{3})+\sqrt{5}}$

$=\dfrac{(\sqrt{2}+\sqrt{3}+\sqrt{5})-(\sqrt{2}+\sqrt{3}-\sqrt{5})}{(\sqrt{2}+\sqrt{3})^2-(\sqrt{5})^2}$

$=\dfrac{2\sqrt{5}}{2\sqrt{6}}=\dfrac{\sqrt{5}}{\sqrt{6}}=\dfrac{\sqrt{30}}{6}$

$\boxed{11}$ 次の値を求めよ。

(1) 49 の平方根　(2) $\sqrt{49}$　(3) $-\sqrt{49}$

解答 (1) 7 と -7　(2) 7　(3) -7

解説

(1) $7^2=(-7)^2=49$ であるから，49 の平方根は　7 と -7

(2) $\sqrt{49}=\sqrt{7^2}=7$

(3) $-\sqrt{49}=-\sqrt{7^2}=-7$

$\boxed{12}$ 次の値を求めよ。

(1) $\sqrt{4^2}$　(2) $\sqrt{(-5)^2}$　(3) $\sqrt{(-8)(-2)}$

解答 (1) 4　(2) 5　(3) 4

解説

(1) $\sqrt{4^2}=4$

(2) $\sqrt{(-5)^2}=-(-5)=5$

(3) $\sqrt{(-8)(-2)}=\sqrt{16}=\sqrt{4^2}=4$

$\boxed{13}$ 次の式を計算せよ。

(1) $6\sqrt{2}+5\sqrt{2}-3\sqrt{2}$　(2) $3\sqrt{5}-4\sqrt{5}+2\sqrt{5}$

(3) $\sqrt{8}\sqrt{14}$　(4) $\sqrt{15}\sqrt{35}\sqrt{42}$　(5) $\dfrac{\sqrt{12}\sqrt{20}}{\sqrt{15}}$

(6) $\sqrt{3}(\sqrt{12}-\sqrt{24})$　(7) $(\sqrt{7}+\sqrt{5})^2$

解答 (1) $8\sqrt{2}$　(2) $\sqrt{5}$　(3) $4\sqrt{7}$　(4) $105\sqrt{2}$　(5) 4　(6) $6-6\sqrt{2}$

(7) $12+2\sqrt{35}$

解説

(1) $6\sqrt{2}+5\sqrt{2}-3\sqrt{2}=(6+5-3)\sqrt{2}=8\sqrt{2}$

(2) $3\sqrt{5}-4\sqrt{5}+2\sqrt{5}=(3-4+2)\sqrt{5}=\sqrt{5}$

(3) $\sqrt{8}\sqrt{14}=\sqrt{8\times14}=\sqrt{2^3\times2\cdot7}=\sqrt{2^4\cdot7}=\sqrt{4^2\cdot7}=4\sqrt{7}$

(4) $\sqrt{15}\sqrt{35}\sqrt{42}=\sqrt{15\times35\times42}=\sqrt{2\cdot3^2\cdot5^2\cdot7^2}$

$=3\cdot5\cdot7\sqrt{2}=105\sqrt{2}$

(5) $\dfrac{\sqrt{12}\sqrt{20}}{\sqrt{15}}=\dfrac{\sqrt{12\times20}}{\sqrt{15}}=\sqrt{\dfrac{12\times20}{15}}=\sqrt{4^2}=4$

(6) $\sqrt{3}(\sqrt{12}-\sqrt{24})=\sqrt{3}(\sqrt{2^2\cdot3}-\sqrt{2^2\cdot6})=\sqrt{3}(2\sqrt{3}-2\sqrt{6})$

$=2(\sqrt{3})^2-2\sqrt{3}\sqrt{6}=2\cdot3-2\sqrt{3\times6}$

$=6-2\sqrt{2\cdot3^2}=6-6\sqrt{2}$

(7) $(\sqrt{7}+\sqrt{5})^2=(\sqrt{7})^2+2\sqrt{7}\sqrt{5}+(\sqrt{5})^2=7+2\sqrt{7\times5}+5$

$=12+2\sqrt{35}$

$\boxed{14}$ 次の式を，分母を有理化して簡単にせよ。

(1) $\dfrac{12}{\sqrt{5}}$　(2) $\dfrac{4}{3\sqrt{8}}$　(3) $\dfrac{5}{\sqrt{5}-\sqrt{3}}$

解答 (1) $\dfrac{12\sqrt{5}}{5}$　(2) $\dfrac{\sqrt{2}}{3}$　(3) $\dfrac{5(\sqrt{5}+\sqrt{3})}{2}$

解説

(1) $\dfrac{12}{\sqrt{5}}=\dfrac{12\times\sqrt{5}}{\sqrt{5}\times\sqrt{5}}=\dfrac{12\sqrt{5}}{5}$

(2) $\dfrac{4}{3\sqrt{8}}=\dfrac{4}{3\cdot2\sqrt{2}}=\dfrac{4\times\sqrt{2}}{3\cdot2\sqrt{2}\times\sqrt{2}}=\dfrac{4\sqrt{2}}{3\cdot4}=\dfrac{\sqrt{2}}{3}$

(3) $\dfrac{5}{\sqrt{5}-\sqrt{3}}=\dfrac{5(\sqrt{5}+\sqrt{3})}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})}=\dfrac{5(\sqrt{5}+\sqrt{3})}{5-3}$

$=\dfrac{5(\sqrt{5}+\sqrt{3})}{2}$

$\boxed{15}$ (1)～(4) の式を計算せよ。また，(5) について答えよ。

(1) $\sqrt{200}-3\sqrt{18}+\sqrt{50}$　(2) $(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})$

(3) $(3\sqrt{2}-2\sqrt{3})(\sqrt{2}+4\sqrt{3})$　(4) $(\sqrt{2}+\sqrt{3}+\sqrt{5})(\sqrt{2}+\sqrt{3}-\sqrt{5})$

(5) $a<0$，$b>0$ のとき，$\sqrt{a^6b^2}$ を根号がつかない形で表せ。

解答 (1) $6\sqrt{2}$　(2) 2　(3) $-18+10\sqrt{6}$　(4) $2\sqrt{6}$　(5) $-a^3b$

解説

(1) $\sqrt{200}-3\sqrt{18}+\sqrt{50}=\sqrt{10^2\cdot2}-3\sqrt{3^2\cdot2}+\sqrt{5^2\cdot2}=10\sqrt{2}-9\sqrt{2}+5\sqrt{2}$

$=(10-9+5)\sqrt{2}=6\sqrt{2}$

(2) $(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})=(\sqrt{5})^2-(\sqrt{3})^2=5-3=2$

(3) $(3\sqrt{2}-2\sqrt{3})(\sqrt{2}+4\sqrt{3})=3(\sqrt{2})^2+(12-2)\sqrt{2}\sqrt{3}-8(\sqrt{3})^2$

$=3\cdot2+10\sqrt{6}-8\cdot3=-18+10\sqrt{6}$

(4) $\{(\sqrt{2}+\sqrt{3})+\sqrt{5}\}\{(\sqrt{2}+\sqrt{3})-\sqrt{5}\}=(\sqrt{2}+\sqrt{3})^2-(\sqrt{5})^2$

$=(\sqrt{2})^2+2\sqrt{2}\sqrt{3}+(\sqrt{3})^2-5$

$=2+2\sqrt{6}+3-5=2\sqrt{6}$

(5) $a^6b^2=(a^3b)^2$ で，$a<0$，$b>0$ であるから　$a^3b<0$

よって　$\sqrt{a^6b^2}=\sqrt{(a^3b)^2}=|a^3b|=-a^3b$

$\boxed{16}$ 次の式を，分母を有理化して簡単にせよ。

(1) $\dfrac{\sqrt{5}}{\sqrt{3}+1}-\dfrac{\sqrt{3}}{\sqrt{5}-\sqrt{3}}$　(2) $\dfrac{\sqrt{2}-\sqrt{3}-\sqrt{5}}{\sqrt{2}+\sqrt{3}+\sqrt{5}}$

解答 (1) $-\dfrac{\sqrt{5}+3}{2}$　(2) $\dfrac{\sqrt{6}-\sqrt{15}}{3}$

解説

(1) (与式)$=\dfrac{\sqrt{5}(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)}-\dfrac{\sqrt{3}(\sqrt{5}+\sqrt{3})}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})}$

$=\dfrac{\sqrt{15}-\sqrt{5}}{3-1}-\dfrac{\sqrt{15}+3}{5-3}=-\dfrac{\sqrt{5}+3}{2}$

(2) (与式)$=\dfrac{(\sqrt{2}-\sqrt{3}-\sqrt{5})\{(\sqrt{2}+\sqrt{3})-\sqrt{5}\}}{\{(\sqrt{2}+\sqrt{3})+\sqrt{5}\}\{(\sqrt{2}+\sqrt{3})-\sqrt{5}\}}$

$=\dfrac{\{(\sqrt{2}-\sqrt{5})-\sqrt{3}\}\{(\sqrt{2}-\sqrt{5})+\sqrt{3}\}}{(\sqrt{2}+\sqrt{3})^2-(\sqrt{5})^2}$

$$=\frac{(\sqrt{2}-\sqrt{5})^2-(\sqrt{3})^2}{5+2\sqrt{6}-5}=\frac{4-2\sqrt{10}}{2\sqrt{6}}=\frac{2-\sqrt{10}}{\sqrt{6}}$$
$$=\frac{(2-\sqrt{10})\cdot\sqrt{6}}{\sqrt{6}\cdot\sqrt{6}}=\frac{2\sqrt{6}-2\sqrt{15}}{6}=\frac{\sqrt{6}-\sqrt{15}}{3}$$

17 次の式を，分母を有理化して簡単にせよ。

(1) $\frac{3\sqrt{2}}{2\sqrt{3}}-\frac{\sqrt{3}}{3\sqrt{2}}+\frac{1}{2\sqrt{6}}$　(2) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}+\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$

(3) $\frac{1}{1-\sqrt{2}}+\frac{1}{1+\sqrt{2}}-\frac{1}{2-\sqrt{2}}-\frac{1}{2+\sqrt{2}}$　(4) $\frac{3-\sqrt{3}+\sqrt{6}}{3+\sqrt{3}+\sqrt{6}}$

解答 (1) $\frac{5\sqrt{6}}{12}$　(2) $9-2\sqrt{6}+\sqrt{15}$　(3) -4　(4) $\frac{-\sqrt{2}+\sqrt{6}}{2}$

解説

(1) $\frac{3\sqrt{2}}{2\sqrt{3}}-\frac{\sqrt{3}}{3\sqrt{2}}+\frac{1}{2\sqrt{6}}=\frac{3\sqrt{2}\times\sqrt{3}}{2\sqrt{3}\times\sqrt{3}}-\frac{\sqrt{3}\times\sqrt{2}}{3\sqrt{2}\times\sqrt{2}}+\frac{\sqrt{6}}{2\sqrt{6}\times\sqrt{6}}$
$=\frac{3\sqrt{6}}{6}-\frac{\sqrt{6}}{6}+\frac{\sqrt{6}}{12}=\frac{6\sqrt{6}-2\sqrt{6}+\sqrt{6}}{12}$
$=\frac{5\sqrt{6}}{12}$

(2) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}+\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}=\frac{(\sqrt{3}-\sqrt{2})^2}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}+\frac{(\sqrt{5}+\sqrt{3})^2}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})}$
$=\frac{3-2\sqrt{6}+2}{3-2}+\frac{5+2\sqrt{15}+3}{5-3}$
$=5-2\sqrt{6}+\frac{8+2\sqrt{15}}{2}$
$=5-2\sqrt{6}+4+\sqrt{15}$
$=9-2\sqrt{6}+\sqrt{15}$

(3) $\frac{1}{1-\sqrt{2}}+\frac{1}{1+\sqrt{2}}-\frac{1}{2-\sqrt{2}}-\frac{1}{2+\sqrt{2}}$
$=\frac{(1+\sqrt{2})+(1-\sqrt{2})}{(1-\sqrt{2})(1+\sqrt{2})}-\frac{(2+\sqrt{2})+(2-\sqrt{2})}{(2-\sqrt{2})(2+\sqrt{2})}$
$=\frac{2}{1-2}-\frac{4}{4-2}=-4$

(4) $\frac{3-\sqrt{3}+\sqrt{6}}{3+\sqrt{3}+\sqrt{6}}=\frac{(3-\sqrt{3}+\sqrt{6})\{3-(\sqrt{3}+\sqrt{6})\}}{\{3+(\sqrt{3}+\sqrt{6})\}\{3-(\sqrt{3}+\sqrt{6})\}}$
$=\frac{\{(3-\sqrt{3})+\sqrt{6}\}\{(3-\sqrt{3})-\sqrt{6}\}}{\{3+(\sqrt{3}+\sqrt{6})\}\{3-(\sqrt{3}+\sqrt{6})\}}$
$=\frac{(3-\sqrt{3})^2-(\sqrt{6})^2}{3^2-(\sqrt{3}+\sqrt{6})^2}=\frac{(12-6\sqrt{3})-6}{9-(9+6\sqrt{2})}=\frac{6-6\sqrt{3}}{-6\sqrt{2}}$
$=\frac{-1+\sqrt{3}}{\sqrt{2}}=\frac{(-1+\sqrt{3})\sqrt{2}}{(\sqrt{2})^2}=\frac{-\sqrt{2}+\sqrt{6}}{2}$

18 (1) 2乗すると7になる数を求めよ。　(2) 10の平方根を求めよ。

(3) $\sqrt{36}$ の値を求めよ。　(4) $-\sqrt{64}$ の値を求めよ。

解答 (1) $\pm\sqrt{7}$　(2) $\pm\sqrt{10}$　(3) 6　(4) -8

解説

(1) 2乗すると7になる数は7の平方根であるから　$\pm\sqrt{7}$
(2) $\pm\sqrt{10}$
(3) $\sqrt{36}=6$
(4) $-\sqrt{64}=-8$

19 次の値を求めよ。

(1) $\sqrt{27}$　(2) $\sqrt{72}$　(3) $\sqrt{0.0049}$　(4) $\sqrt{\frac{75}{16}}$

(5) $(-\sqrt{5})^2$　(6) $\sqrt{(-8)^2}$　(7) $-\sqrt{(-3)^2}$　(8) $\sqrt{(-8)(-2)}$

(9) $\sqrt{5}\sqrt{20}$　(10) $\sqrt{12}\sqrt{18}$　(11) $\frac{\sqrt{24}}{\sqrt{3}}$　(12) $\frac{\sqrt{108}}{\sqrt{6}}$

解答 (1) $3\sqrt{3}$　(2) $6\sqrt{2}$　(3) 0.07　(4) $\frac{5\sqrt{3}}{4}$　(5) 5　(6) 8
(7) -3　(8) 4　(9) 10　(10) $6\sqrt{6}$　(11) $2\sqrt{2}$　(12) $3\sqrt{2}$

解説

(1) $\sqrt{27}=\sqrt{3^2\cdot3}=3\sqrt{3}$
(2) $\sqrt{72}=\sqrt{6^2\cdot2}=6\sqrt{2}$
(3) $\sqrt{0.0049}=\sqrt{0.07^2}=0.07$
(4) $\sqrt{\frac{75}{16}}=\sqrt{\frac{5^2\cdot3}{4^2}}=\frac{\sqrt{5^2\cdot3}}{\sqrt{4^2}}=\frac{5\sqrt{3}}{4}$
(5) $(-\sqrt{5})^2=5$
(6) $\sqrt{(-8)^2}=\sqrt{64}=\sqrt{8^2}=8$
(7) $-\sqrt{(-3)^2}=-\sqrt{9}=-\sqrt{3^2}=-3$
(8) $\sqrt{(-8)(-2)}=\sqrt{16}=\sqrt{4^2}=4$
(9) $\sqrt{5}\sqrt{20}=\sqrt{5\cdot20}=\sqrt{10^2}=10$
(10) $\sqrt{12}\sqrt{18}=\sqrt{12\cdot18}=\sqrt{2\cdot6\cdot3\cdot6}=\sqrt{6^2\cdot6}=6\sqrt{6}$
(11) $\frac{\sqrt{24}}{\sqrt{3}}=\sqrt{\frac{24}{3}}=\sqrt{8}=\sqrt{2^2\cdot2}=2\sqrt{2}$
(12) $\frac{\sqrt{108}}{\sqrt{6}}=\sqrt{\frac{108}{6}}=\sqrt{18}=\sqrt{3^2\cdot2}=3\sqrt{2}$

20 次の式を計算せよ。

(1) $3\sqrt{3}+\sqrt{75}-\sqrt{48}$　(2) $2\sqrt{7}-\sqrt{63}+\sqrt{28}$　(3) $\sqrt{3}(2\sqrt{3}-\sqrt{6})$

(4) $\sqrt{5}(3\sqrt{10}-2\sqrt{5})$　(5) $(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})$　(6) $(\sqrt{20}+\sqrt{3})(\sqrt{5}-\sqrt{27})$

(7) $(\sqrt{3}+\sqrt{5})^2$　(8) $(2\sqrt{3}-3\sqrt{2})^2$

解答 (1) $4\sqrt{3}$　(2) $\sqrt{7}$　(3) $6-3\sqrt{2}$　(4) $15\sqrt{2}-10$　(5) 2
(6) $1-5\sqrt{15}$　(7) $8+2\sqrt{15}$　(8) $30-12\sqrt{6}$

解説

(1) 与式$=3\sqrt{3}+5\sqrt{3}-4\sqrt{3}=4\sqrt{3}$
(2) 与式$=2\sqrt{7}-3\sqrt{7}+2\sqrt{7}=\sqrt{7}$
(3) 与式$=\sqrt{3}\cdot2\sqrt{3}-\sqrt{3}\cdot\sqrt{6}=2\cdot3-\sqrt{3^2\cdot2}=6-3\sqrt{2}$
(4) 与式$=\sqrt{5}\cdot3\sqrt{10}-\sqrt{5}\cdot2\sqrt{5}=3\cdot5\sqrt{2}-2\cdot5=15\sqrt{2}-10$
(5) 与式$=(\sqrt{5})^2-(\sqrt{3})^2=5-3=2$
(6) 与式$=(2\sqrt{5}+\sqrt{3})(\sqrt{5}-3\sqrt{3})=2\sqrt{5}\cdot\sqrt{5}-2\sqrt{5}\cdot3\sqrt{3}+\sqrt{3}\cdot\sqrt{5}-\sqrt{3}\cdot3\sqrt{3}$
$=10-6\sqrt{15}+\sqrt{15}-9=1-5\sqrt{15}$
(7) 与式$=(\sqrt{3})^2+2\cdot\sqrt{3}\cdot\sqrt{5}+(\sqrt{5})^2=3+2\sqrt{15}+5=8+2\sqrt{15}$
(8) 与式$=(2\sqrt{3})^2-2\cdot2\sqrt{3}\cdot3\sqrt{2}+(3\sqrt{2})^2=12-12\sqrt{6}+18=30-12\sqrt{6}$

21 次の式の分母を有理化せよ。

(1) $\frac{2}{\sqrt{5}}$　(2) $\frac{4}{3\sqrt{8}}$　(3) $\frac{1}{\sqrt{2}+1}$　(4) $\frac{1}{\sqrt{6}-\sqrt{3}}$

(5) $\frac{2+\sqrt{3}}{2-\sqrt{3}}$　(6) $\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}$　(7) $\frac{\sqrt{3}-1}{2\sqrt{3}-5}$　(8) $\frac{2\sqrt{2}-\sqrt{3}}{\sqrt{3}+\sqrt{2}}$

解答 (1) $\frac{2\sqrt{5}}{5}$　(2) $\frac{\sqrt{2}}{3}$　(3) $\sqrt{2}-1$　(4) $\frac{\sqrt{6}+\sqrt{3}}{3}$　(5) $7+4\sqrt{3}$
(6) $\frac{7-2\sqrt{10}}{3}$　(7) $-\frac{1+3\sqrt{3}}{13}$　(8) $-7+3\sqrt{6}$

解説

(1) 与式$=\frac{2\times\sqrt{5}}{\sqrt{5}\times\sqrt{5}}=\frac{2\sqrt{5}}{5}$
(2) 与式$=\frac{4}{3\cdot2\sqrt{2}}=\frac{2}{3\sqrt{2}}=\frac{2\times\sqrt{2}}{3\sqrt{2}\times\sqrt{2}}=\frac{2\sqrt{2}}{6}=\frac{\sqrt{2}}{3}$
(3) 与式$=\frac{\sqrt{2}-1}{(\sqrt{2}+1)(\sqrt{2}-1)}=\frac{\sqrt{2}-1}{(\sqrt{2})^2-1^2}=\sqrt{2}-1$
(4) 与式$=\frac{\sqrt{6}+\sqrt{3}}{(\sqrt{6}-\sqrt{3})(\sqrt{6}+\sqrt{3})}=\frac{\sqrt{6}+\sqrt{3}}{(\sqrt{6})^2-(\sqrt{3})^2}=\frac{\sqrt{6}+\sqrt{3}}{3}$
(5) 与式$=\frac{(2+\sqrt{3})^2}{(2-\sqrt{3})(2+\sqrt{3})}=\frac{4+4\sqrt{3}+3}{2^2-(\sqrt{3})^2}=7+4\sqrt{3}$
(6) 与式$=\frac{(\sqrt{5}-\sqrt{2})^2}{(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})}=\frac{5-2\sqrt{10}+2}{(\sqrt{5})^2-(\sqrt{2})^2}=\frac{7-2\sqrt{10}}{3}$
(7) 与式$=\frac{(\sqrt{3}-1)(2\sqrt{3}+5)}{(2\sqrt{3}-5)(2\sqrt{3}+5)}=\frac{6+5\sqrt{3}-2\sqrt{3}-5}{(2\sqrt{3})^2-5^2}=-\frac{1+3\sqrt{3}}{13}$
(8) 与式$=\frac{(2\sqrt{2}-\sqrt{3})(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}=\frac{2\sqrt{6}-4-3+\sqrt{6}}{(\sqrt{3})^2-(\sqrt{2})^2}=-7+3\sqrt{6}$

22 次の式を計算せよ。

(1) $(1+\sqrt{2}-\sqrt{3})^2$　(2) $(3-\sqrt{2}-\sqrt{11})(3-\sqrt{2}+\sqrt{11})$

解答 (1) $6+2\sqrt{2}-2\sqrt{3}-2\sqrt{6}$　(2) $-6\sqrt{2}$

解説

(1) 与式$=1^2+(\sqrt{2})^2+(-\sqrt{3})^2+2\cdot1\cdot\sqrt{2}+2\cdot\sqrt{2}\cdot(-\sqrt{3})+2\cdot(-\sqrt{3})\cdot1$
$=1+2+3+2\sqrt{2}-2\sqrt{6}-2\sqrt{3}=6+2\sqrt{2}-2\sqrt{3}-2\sqrt{6}$

別解 与式$=\{(1+\sqrt{2})-\sqrt{3}\}^2=(1+\sqrt{2})^2-2(1+\sqrt{2})\sqrt{3}+(\sqrt{3})^2$
$=(1+2\sqrt{2}+2)-2\sqrt{3}-2\sqrt{6}+3=6+2\sqrt{2}-2\sqrt{3}-2\sqrt{6}$

(2) 与式$=\{(3-\sqrt{2})-\sqrt{11}\}\{(3-\sqrt{2})+\sqrt{11}\}=(3-\sqrt{2})^2-(\sqrt{11})^2$
$=9-6\sqrt{2}+2-11=-6\sqrt{2}$

23 次の式を計算せよ。

(1) $\frac{3\sqrt{5}-5\sqrt{3}}{\sqrt{5}+\sqrt{3}}+\frac{3\sqrt{5}+4\sqrt{3}}{3\sqrt{5}-4\sqrt{3}}$　(2) $\frac{\sqrt{2}-1}{\sqrt{2}+1}+\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}+\frac{\sqrt{3}+\sqrt{2}}{2-\sqrt{3}}$

解答 (1) $-16-12\sqrt{15}$　(2) $11+2\sqrt{3}-\sqrt{6}$

解説

(1) 与式$=\dfrac{(3\sqrt{5}-5\sqrt{3})(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})}+\dfrac{(3\sqrt{5}+4\sqrt{3})^2}{(3\sqrt{5}-4\sqrt{3})(3\sqrt{5}+4\sqrt{3})}$

$=\dfrac{15-3\sqrt{15}-5\sqrt{15}+15}{5-3}+\dfrac{45+24\sqrt{15}+48}{45-48}$

$=\dfrac{30-8\sqrt{15}}{2}-\dfrac{93+24\sqrt{15}}{3}=(15-4\sqrt{15})-(31+8\sqrt{15})$

$=-16-12\sqrt{15}$

(2) 与式$=\dfrac{(\sqrt{2}-1)^2}{(\sqrt{2}+1)(\sqrt{2}-1)}+\dfrac{(\sqrt{3}-\sqrt{2})^2}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}+\dfrac{(\sqrt{3}+\sqrt{2})(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})}$

$=(3-2\sqrt{2})+(5-2\sqrt{6})+(2\sqrt{3}+3+2\sqrt{2}+\sqrt{6})$

$=11+2\sqrt{3}-\sqrt{6}$

24 次の式を計算せよ。

(1) $\dfrac{1}{1+\sqrt{2}-\sqrt{3}}$　(2) $\dfrac{\sqrt{5}+\sqrt{3}+\sqrt{2}}{\sqrt{5}+\sqrt{3}-\sqrt{2}}$

(3) $\dfrac{\sqrt{2}+\sqrt{5}+\sqrt{7}}{\sqrt{2}+\sqrt{5}-\sqrt{7}}+\dfrac{\sqrt{2}-\sqrt{5}+\sqrt{7}}{\sqrt{2}-\sqrt{5}-\sqrt{7}}$

解答 (1) $\dfrac{2+\sqrt{2}+\sqrt{6}}{4}$　(2) $\dfrac{\sqrt{6}+\sqrt{15}}{3}$　(3) $2+\sqrt{14}$

解説

(1) 与式$=\dfrac{1+\sqrt{2}+\sqrt{3}}{(1+\sqrt{2}-\sqrt{3})(1+\sqrt{2}+\sqrt{3})}=\dfrac{1+\sqrt{2}+\sqrt{3}}{(1+\sqrt{2})^2-(\sqrt{3})^2}=\dfrac{1+\sqrt{2}+\sqrt{3}}{2\sqrt{2}}$

$=\dfrac{(1+\sqrt{2}+\sqrt{3})\sqrt{2}}{2\sqrt{2}\cdot\sqrt{2}}=\dfrac{2+\sqrt{2}+\sqrt{6}}{4}$

(2) 与式$=\dfrac{(\sqrt{5}+\sqrt{3}+\sqrt{2})(\sqrt{5}-\sqrt{3}+\sqrt{2})}{(\sqrt{5}+\sqrt{3}-\sqrt{2})(\sqrt{5}-\sqrt{3}+\sqrt{2})}=\dfrac{(\sqrt{5}+\sqrt{2})^2-(\sqrt{3})^2}{(\sqrt{5})^2-(\sqrt{3}-\sqrt{2})^2}$

$=\dfrac{4+2\sqrt{10}}{2\sqrt{6}}=\dfrac{2+\sqrt{10}}{\sqrt{6}}=\dfrac{2\sqrt{6}+\sqrt{60}}{6}=\dfrac{2\sqrt{6}+2\sqrt{15}}{6}=\dfrac{\sqrt{6}+\sqrt{15}}{3}$

(3) 与式$=\dfrac{(\sqrt{2}+\sqrt{5}+\sqrt{7})^2}{(\sqrt{2}+\sqrt{5}-\sqrt{7})(\sqrt{2}+\sqrt{5}+\sqrt{7})}+\dfrac{(\sqrt{2}-\sqrt{5}+\sqrt{7})^2}{(\sqrt{2}-\sqrt{5}-\sqrt{7})(\sqrt{2}-\sqrt{5}+\sqrt{7})}$

$=\dfrac{14+2\sqrt{10}+2\sqrt{35}+2\sqrt{14}}{(\sqrt{2}+\sqrt{5})^2-(\sqrt{7})^2}+\dfrac{14-2\sqrt{10}-2\sqrt{35}+2\sqrt{14}}{(\sqrt{2}-\sqrt{5})^2-(\sqrt{7})^2}$

$=\dfrac{14+2\sqrt{10}+2\sqrt{35}+2\sqrt{14}}{2\sqrt{10}}+\dfrac{14-2\sqrt{10}-2\sqrt{35}+2\sqrt{14}}{-2\sqrt{10}}$

$=\dfrac{4\sqrt{10}+4\sqrt{35}}{2\sqrt{10}}=\dfrac{4\sqrt{10}}{2\sqrt{10}}+\dfrac{4\sqrt{35}}{2\sqrt{10}}=2+2\sqrt{\dfrac{7}{2}}=2+\sqrt{14}$

25 $\dfrac{\sqrt{2}+\sqrt{3}-\sqrt{5}}{\sqrt{2}-\sqrt{3}+\sqrt{5}}$ の分母を有理化せよ。

解答 $\dfrac{\sqrt{15}-\sqrt{6}}{3}$

解説

$\dfrac{\sqrt{2}+\sqrt{3}-\sqrt{5}}{\sqrt{2}-\sqrt{3}+\sqrt{5}}=\dfrac{(\sqrt{2}+\sqrt{3}-\sqrt{5})(\sqrt{2}-\sqrt{3}-\sqrt{5})}{(\sqrt{2}-\sqrt{3}+\sqrt{5})(\sqrt{2}-\sqrt{3}-\sqrt{5})}$

$=\dfrac{(\sqrt{2}-\sqrt{5})^2-(\sqrt{3})^2}{(\sqrt{2}-\sqrt{3})^2-(\sqrt{5})^2}=\dfrac{4-2\sqrt{10}}{-2\sqrt{6}}=-\dfrac{2-\sqrt{10}}{\sqrt{6}}=\dfrac{(\sqrt{10}-2)\sqrt{6}}{\sqrt{6}\cdot\sqrt{6}}$

$=\dfrac{\sqrt{60}-2\sqrt{6}}{6}=\dfrac{2\sqrt{15}-2\sqrt{6}}{6}=\dfrac{\sqrt{15}-\sqrt{6}}{3}$

26 次の値を求めよ。

(1) $\sqrt{81}$　(2) $\sqrt{(-7)^2}$　(3) $\sqrt{(-2)(-8)}$　(4) $-\sqrt{\dfrac{9}{4}}$

解答 (1) 9　(2) 7　(3) 4　(4) $-\dfrac{3}{2}$

解説

(1) $\sqrt{81}=\sqrt{9^2}=9$

(2) $\sqrt{(-7)^2}=\sqrt{49}=\sqrt{7^2}=7$

(3) $\sqrt{(-2)(-8)}=\sqrt{16}=\sqrt{4^2}=4$

(4) $-\sqrt{\dfrac{9}{4}}=-\sqrt{\left(\dfrac{3}{2}\right)^2}=-\dfrac{3}{2}$

27 次の式を計算せよ。

(1) $\sqrt{3}\sqrt{12}$　(2) $\sqrt{15}\sqrt{6}$　(3) $\dfrac{\sqrt{54}}{\sqrt{3}}$　(4) $\dfrac{\sqrt{7}}{\sqrt{28}}$

解答 (1) 6　(2) $3\sqrt{10}$　(3) $3\sqrt{2}$　(4) $\dfrac{1}{2}$

解説

(1) $\sqrt{3}\sqrt{12}=\sqrt{3\cdot12}=\sqrt{36}=\sqrt{6^2}=6$

(2) $\sqrt{15}\sqrt{6}=\sqrt{15\cdot6}=\sqrt{90}=\sqrt{3^2\cdot10}=3\sqrt{10}$

(3) $\dfrac{\sqrt{54}}{\sqrt{3}}=\sqrt{\dfrac{54}{3}}=\sqrt{18}=\sqrt{3^2\cdot2}=3\sqrt{2}$

(4) $\dfrac{\sqrt{7}}{\sqrt{28}}=\sqrt{\dfrac{7}{28}}=\sqrt{\dfrac{1}{4}}=\sqrt{\left(\dfrac{1}{2}\right)^2}=\dfrac{1}{2}$

28 次の式を計算せよ。

(1) $3\sqrt{7}+\sqrt{7}-2\sqrt{7}$　(2) $\sqrt{3}+\sqrt{27}-\sqrt{75}$

(3) $\sqrt{50}-2\sqrt{32}+\sqrt{72}$　(4) $\sqrt{3}(2\sqrt{3}-\sqrt{6})$

(5) $(3\sqrt{5}-2\sqrt{3})(4\sqrt{5}+3\sqrt{3})$　(6) $(\sqrt{7}+\sqrt{2})(\sqrt{7}-\sqrt{2})$

(7) $(\sqrt{5}-\sqrt{10})^2$　(8) $(5\sqrt{2}+2\sqrt{3})^2$

解答 (1) $2\sqrt{7}$　(2) $-\sqrt{3}$　(3) $3\sqrt{2}$　(4) $6-3\sqrt{2}$　(5) $42+\sqrt{15}$
(6) 5　(7) $15-10\sqrt{2}$　(8) $62+20\sqrt{6}$

解説

(1) $3\sqrt{7}+\sqrt{7}-2\sqrt{7}=(3+1-2)\sqrt{7}=2\sqrt{7}$

(2) $\sqrt{3}+\sqrt{27}-\sqrt{75}=\sqrt{3}+\sqrt{3^2\cdot3}-\sqrt{5^2\cdot3}$

$=\sqrt{3}+3\sqrt{3}-5\sqrt{3}$

$=(1+3-5)\sqrt{3}=-\sqrt{3}$

(3) $\sqrt{50}-2\sqrt{32}+\sqrt{72}=\sqrt{5^2\cdot2}-2\sqrt{4^2\cdot2}+\sqrt{6^2\cdot2}$

$=5\sqrt{2}-8\sqrt{2}+6\sqrt{2}$

$=(5-8+6)\sqrt{2}=3\sqrt{2}$

(4) $\sqrt{3}(2\sqrt{3}-\sqrt{6})=2(\sqrt{3})^2-\sqrt{3}\sqrt{6}=2\cdot3-\sqrt{3\cdot6}$

$=6-\sqrt{3^2\cdot2}=6-3\sqrt{2}$

(5) $(3\sqrt{5}-2\sqrt{3})(4\sqrt{5}+3\sqrt{3})=12(\sqrt{5})^2+9\sqrt{15}-8\sqrt{15}-6(\sqrt{3})^2$

$=60+\sqrt{15}-18=42+\sqrt{15}$

(6) $(\sqrt{7}+\sqrt{2})(\sqrt{7}-\sqrt{2})=(\sqrt{7})^2-(\sqrt{2})^2$

$=7-2=5$

(7) $(\sqrt{5}-\sqrt{10})^2=(\sqrt{5})^2-2\sqrt{5}\sqrt{10}+(\sqrt{10})^2=5-2\sqrt{50}+10$

$=5+10-2\sqrt{5^2\cdot2}=15-10\sqrt{2}$

別解 $(\sqrt{5}-\sqrt{10})^2=\{\sqrt{5}(1-\sqrt{2})\}^2=(\sqrt{5})^2(1-\sqrt{2})^2$

$=5(1-2\sqrt{2}+2)=5(3-2\sqrt{2})$

$=15-10\sqrt{2}$

(8) $(5\sqrt{2}+2\sqrt{3})^2=(5\sqrt{2})^2+2\cdot5\sqrt{2}\cdot2\sqrt{3}+(2\sqrt{3})^2$

$=5^2\cdot2+20\sqrt{6}+2^2\cdot3=50+20\sqrt{6}+12$

$=62+20\sqrt{6}$

29 次の式の分母を有理化せよ。

(1) $\dfrac{4}{3\sqrt{2}}$　(2) $\dfrac{2}{\sqrt{3}+1}$　(3) $\dfrac{\sqrt{2}+1}{\sqrt{2}-1}$　(4) $\dfrac{\sqrt{3}}{2-\sqrt{5}}$

解答 (1) $\dfrac{2\sqrt{2}}{3}$　(2) $\sqrt{3}-1$　(3) $3+2\sqrt{2}$　(4) $-2\sqrt{3}-\sqrt{15}$

解説

(1) $\dfrac{4}{3\sqrt{2}}=\dfrac{4\times\sqrt{2}}{3\sqrt{2}\times\sqrt{2}}=\dfrac{4\sqrt{2}}{3\cdot2}=\dfrac{2\sqrt{2}}{3}$

(2) $\dfrac{2}{\sqrt{3}+1}=\dfrac{2(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)}=\dfrac{2(\sqrt{3}-1)}{(\sqrt{3})^2-1^2}$

$=\dfrac{2(\sqrt{3}-1)}{3-1}=\sqrt{3}-1$

(3) $\dfrac{\sqrt{2}+1}{\sqrt{2}-1}=\dfrac{(\sqrt{2}+1)^2}{(\sqrt{2}-1)(\sqrt{2}+1)}=\dfrac{2+2\sqrt{2}+1}{(\sqrt{2})^2-1^2}$

$=\dfrac{3+2\sqrt{2}}{2-1}=3+2\sqrt{2}$

(4) $\dfrac{\sqrt{3}}{2-\sqrt{5}}=\dfrac{\sqrt{3}(2+\sqrt{5})}{(2-\sqrt{5})(2+\sqrt{5})}=\dfrac{2\sqrt{3}+\sqrt{15}}{2^2-(\sqrt{5})^2}$

$=\dfrac{2\sqrt{3}+\sqrt{15}}{4-5}=-2\sqrt{3}-\sqrt{15}$

30 (1), (2) の式を計算せよ。また，(3), (4) の式の分母を有理化せよ。

(1) $\sqrt{20}-(\sqrt{45}-4\sqrt{5})$　(2) $(\sqrt{12}-\sqrt{8})(\sqrt{48}+\sqrt{32})$

(3) $\dfrac{1}{\sqrt{5}-\sqrt{3}}$　(4) $\dfrac{2-\sqrt{3}}{2+\sqrt{3}}$

解答 (1) $3\sqrt{5}$　(2) 8　(3) $\dfrac{\sqrt{5}+\sqrt{3}}{2}$　(4) $7-4\sqrt{3}$

解説

(1) $\sqrt{20}-(\sqrt{45}-4\sqrt{5})=\sqrt{2^2\cdot5}-\sqrt{3^2\cdot5}+4\sqrt{5}$

$=2\sqrt{5}-3\sqrt{5}+4\sqrt{5}$

$=(2-3+4)\sqrt{5}=3\sqrt{5}$

(2) $(\sqrt{12}-\sqrt{8})(\sqrt{48}+\sqrt{32})=(\sqrt{2^2\cdot3}-\sqrt{2^2\cdot2})(\sqrt{4^2\cdot3}+\sqrt{4^2\cdot2})$

$=(2\sqrt{3}-2\sqrt{2})(4\sqrt{3}+4\sqrt{2})$

$=2\cdot4(\sqrt{3})^2+\{2\cdot4+(-2)\cdot4\}\sqrt{3}\sqrt{2}+(-2)\cdot4(\sqrt{2})^2$

$=8\cdot3-8\cdot2=8$

別解 $(\sqrt{12}-\sqrt{8})(\sqrt{48}+\sqrt{32})=(2\sqrt{3}-2\sqrt{2})(4\sqrt{3}+4\sqrt{2})$

$=2(\sqrt{3}-\sqrt{2})\cdot4(\sqrt{3}+\sqrt{2})$

$=8(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})$

$=8\{(\sqrt{3})^2-(\sqrt{2})^2\}=8(3-2)=8$

(3) $\dfrac{1}{\sqrt{5}-\sqrt{3}}=\dfrac{\sqrt{5}+\sqrt{3}}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})}=\dfrac{\sqrt{5}+\sqrt{3}}{(\sqrt{5})^2-(\sqrt{3})^2}$

$=\dfrac{\sqrt{5}+\sqrt{3}}{5-3}=\dfrac{\sqrt{5}+\sqrt{3}}{2}$

(4) $\dfrac{2-\sqrt{3}}{2+\sqrt{3}}=\dfrac{(2-\sqrt{3})^2}{(2+\sqrt{3})(2-\sqrt{3})}=\dfrac{4-4\sqrt{3}+3}{2^2-(\sqrt{3})^2}$

$=\dfrac{7-4\sqrt{3}}{4-3}=7-4\sqrt{3}$

31 次の式を計算せよ。

(1) $(1+\sqrt{2}-\sqrt{3})^2$　　(2) $(1+\sqrt{2}-\sqrt{6})(1-\sqrt{2}+\sqrt{6})$

(3) $\dfrac{\sqrt{7}}{\sqrt{7}-\sqrt{5}}-\dfrac{\sqrt{5}}{\sqrt{7}+\sqrt{5}}$　　(4) $\dfrac{1}{2+\sqrt{5}}+\dfrac{\sqrt{5}-3}{\sqrt{5}+1}$

解答 (1) $6+2\sqrt{2}-2\sqrt{3}-2\sqrt{6}$　(2) $4\sqrt{3}-7$　(3) 6　(4) 0

解説

(1) $(1+\sqrt{2}-\sqrt{3})^2=\{(1+\sqrt{2})-\sqrt{3}\}^2$

$=(1+\sqrt{2})^2-2\cdot(1+\sqrt{2})\cdot\sqrt{3}+(\sqrt{3})^2$

$=(3+2\sqrt{2})-2\sqrt{3}-2\sqrt{6}+3$

$=6+2\sqrt{2}-2\sqrt{3}-2\sqrt{6}$

別解 $(1+\sqrt{2}-\sqrt{3})^2=1^2+(\sqrt{2})^2+(-\sqrt{3})^2+2\cdot1\cdot\sqrt{2}+2\cdot\sqrt{2}\cdot(-\sqrt{3})+2\cdot(-\sqrt{3})\cdot1$

$=1+2+3+2\sqrt{2}-2\sqrt{6}-2\sqrt{3}$

$=6+2\sqrt{2}-2\sqrt{3}-2\sqrt{6}$

(2) $(1+\sqrt{2}-\sqrt{6})(1-\sqrt{2}+\sqrt{6})=\{1+(\sqrt{2}-\sqrt{6})\}\{1-(\sqrt{2}-\sqrt{6})\}$

$=1^2-(\sqrt{2}-\sqrt{6})^2=1-(8-2\sqrt{12})$

$=2\sqrt{12}-7=4\sqrt{3}-7$

(3) $\dfrac{\sqrt{7}}{\sqrt{7}-\sqrt{5}}-\dfrac{\sqrt{5}}{\sqrt{7}+\sqrt{5}}=\dfrac{\sqrt{7}(\sqrt{7}+\sqrt{5})}{(\sqrt{7}-\sqrt{5})(\sqrt{7}+\sqrt{5})}-\dfrac{\sqrt{5}(\sqrt{7}-\sqrt{5})}{(\sqrt{7}+\sqrt{5})(\sqrt{7}-\sqrt{5})}$

$=\dfrac{7+\sqrt{35}}{7-5}-\dfrac{\sqrt{35}-5}{7-5}$

$=\dfrac{7+\sqrt{35}}{2}-\dfrac{\sqrt{35}-5}{2}=\dfrac{12}{2}=6$

(4) $\dfrac{1}{2+\sqrt{5}}+\dfrac{\sqrt{5}-3}{\sqrt{5}+1}=\dfrac{2-\sqrt{5}}{(2+\sqrt{5})(2-\sqrt{5})}+\dfrac{(\sqrt{5}-3)(\sqrt{5}-1)}{(\sqrt{5}+1)(\sqrt{5}-1)}$

$=\dfrac{2-\sqrt{5}}{4-5}+\dfrac{5-4\sqrt{5}+3}{5-1}$

$=-(2-\sqrt{5})+\dfrac{8-4\sqrt{5}}{4}$

$=-(2-\sqrt{5})+(2-\sqrt{5})=0$

32 次の式の分母を有理化せよ。

(1) $\dfrac{1}{1+\sqrt{5}-\sqrt{6}}$　　(2) $\dfrac{1-\sqrt{2}+\sqrt{3}}{1+\sqrt{2}+\sqrt{3}}$　　(3) $\dfrac{1}{\sqrt{2}+\sqrt{3}-\sqrt{5}}$

解答 (1) $\dfrac{\sqrt{5}+5+\sqrt{30}}{10}$　(2) $\sqrt{3}-\sqrt{2}$　(3) $\dfrac{2\sqrt{3}+3\sqrt{2}+\sqrt{30}}{12}$

解説

(1) 与式$=\dfrac{1+\sqrt{5}+\sqrt{6}}{(1+\sqrt{5}-\sqrt{6})(1+\sqrt{5}+\sqrt{6})}=\dfrac{1+\sqrt{5}+\sqrt{6}}{(1+\sqrt{5})^2-(\sqrt{6})^2}$

$=\dfrac{1+\sqrt{5}+\sqrt{6}}{2\sqrt{5}}=\dfrac{(1+\sqrt{5}+\sqrt{6})\sqrt{5}}{2(\sqrt{5})^2}$

$=\dfrac{\sqrt{5}+5+\sqrt{30}}{10}$

(2) 与式$=\dfrac{(1-\sqrt{2}+\sqrt{3})(1+\sqrt{2}-\sqrt{3})}{(1+\sqrt{2}+\sqrt{3})(1+\sqrt{2}-\sqrt{3})}=\dfrac{\{1-(\sqrt{2}-\sqrt{3})\}\{1+(\sqrt{2}-\sqrt{3})\}}{(1+\sqrt{2})^2-(\sqrt{3})^2}$

$=\dfrac{1^2-(\sqrt{2}-\sqrt{3})^2}{2\sqrt{2}}=\dfrac{2\sqrt{6}-4}{2\sqrt{2}}=\dfrac{\sqrt{6}-2}{\sqrt{2}}$

$=\dfrac{(\sqrt{6}-2)\sqrt{2}}{(\sqrt{2})^2}=\dfrac{2\sqrt{3}-2\sqrt{2}}{2}=\sqrt{3}-\sqrt{2}$

(3) 与式$=\dfrac{\sqrt{2}+\sqrt{3}+\sqrt{5}}{(\sqrt{2}+\sqrt{3}-\sqrt{5})(\sqrt{2}+\sqrt{3}+\sqrt{5})}=\dfrac{\sqrt{2}+\sqrt{3}+\sqrt{5}}{(\sqrt{2}+\sqrt{3})^2-(\sqrt{5})^2}$

$=\dfrac{\sqrt{2}+\sqrt{3}+\sqrt{5}}{2\sqrt{6}}=\dfrac{(\sqrt{2}+\sqrt{3}+\sqrt{5})\sqrt{6}}{2(\sqrt{6})^2}$

$=\dfrac{2\sqrt{3}+3\sqrt{2}+\sqrt{30}}{12}$